Solving Macroeconomic Problems and Exercises

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As we approach the end of the course (and the exam), it feels like it is time for a review—a practical review, a quick reminder on how to use the models of the macroeconomy presented in this course to understand concrete situations and to solve problems. So here it is: a quick review of how to solve six kinds of problems: growth-model problems, flexible-price-model problems, money-and-inflation (under flexible prices) problems, income-and-expenditure sticky-price-model problems, IS-curve sticky-price-model problems, and Phillips-curve-and-expectations problems.

The Growth Model

The keys to carrying out practically any analysis using the long-run growth model are threefold. One needs to be able to calculate (a) the economy's balanced-growth capital-output ratio κ^*, the economy's level of the efficiency of labor E, and—last—how fast the economy is converging to its balanced-growth capital-output ratio.

Calculating the economy's balanced-growth capital-output ratio κ* is straightforward. It is:

$$\kappa^* = \frac{s}{n + g + \delta}$$

The balanced-growth capital-output ratio κ* is equal to the economy's savings-investment rate s divided by the sum of the labor force growth rate n, the efficiency of labor growth rate g, and the depreciation rate δ.

The level of the efficiency of labor at any time t is straightforward to calculate. Let E_0 stand for the level of the efficiency of labor at some time when it is known. Then E_t, the level of the efficiency of labor some t years later, is simply:

$$E_t = E_0 \times (1 + g)^t$$

where g is the annual growth rate of the efficiency of labor. Once both the balanced-growth capital-output ratio and the efficiency of labor are known, it is straightforward to calculate the balanced-growth-path level of output per worker for the economy. The balanced growth path level of output per worker is given by:

$$\left(\frac{Y_t}{L_t}\right)_{bg} = (\kappa^*)^{\lambda} \times E_t$$

with λ being the growth multiplier, and with $\lambda = (\alpha/(1-\alpha))$, where α is the diminishing-returns-to-capital parameter in the production function.
What if the economy is not on its balanced-growth path? Then it will get to it soon. Every year the capital-output ratio closes a fraction equal to $(1-\alpha)(n+g+\delta)$ of the gap between its current value and its balanced-growth value.

To recap, calculating output per worker when the economy is on its balanced-growth path is a simple three-step procedure:

- First, calculate the balanced-growth capital-output ratio.
- Second, amplify the balanced-growth capital-output ratio κ^* by raising it to the power of the growth multiplier λ.
- Third, multiply the result by the current value of the efficiency of labor E_t.

What if the economy is not on its balanced-growth path? makes analyzing the long-run growth of an economy relatively easily as well:

1. First calculate the balanced-growth path.

µ Predict that the economy is heading from its current position to its balanced-growth path.

The Flexible-Price Model

If the key to the growth model is the balanced-growth capital-output ratio κ^*, the key to the flexible-price model is the equilibrium interest rate r: the interest rate that equilibrates the demand and supply of the flow-of-funds through financial markets. The solution to almost any problem posed in the flexible-price model will follow almost immediately once one (a) calculates the equilibrium interest rate r, and (b) applies the equilibrium interest rate r to the behavioral equations of the flexible-price model.

The equilibrium interest rate r is the rate that balances the flow of savings to the flow of investment. In algebra, it is the interest rate that makes the equality:

$$S_p - D - NX = I$$

hold, where S_p is the flow of private savings into financial markets, -D is the negative of the government deficit—it is the government surplus, the flow of public savings into financial markets, -NX is the net flow of foreign savings into financial markets (for the negative of net exports, -NX, is equal to the net flow of foreign savings into domestic financial markets, and I is the flow of investment spending.

To consider these four terms one at a time, investment spending is:

$$I = I_0 - I_r \times r$$

It depends on the baseline level of investment I_0, on the interest sensitivity of investment I_r, and on the interest rate r.

The capital inflow—minus net exports—is:

$$-NX = IM_y Y - \left(X_f Y^f + X_\varepsilon \left(\varepsilon_0 - \varepsilon_r \left(r - r^f\right)\right)\right)$$

Minus net exports is equal to imports minus exports. Imports are equal to IM_y, the propensity to import, times the level of output Y (and remember that in the flexible-price model the level of output Y is always equal to potential output Y*). Exports are equal to (a) a term equal to foreign income Y^f times X_f, the foreign propensity to buy domestic exports; plus the sensitivity of exports to the exchange rate X_ε times the exchange rate, with the exchange rate equal to its baseline fundamental ε_0 minus its sensitivity to interest rates ε_r times the difference between the home interest rate r and the foreign interest rate r^f.

Private savings are equal to income minus consumption spending minus taxes:

$$S_p = Y - T - C$$

Or, breaking taxes and consumption spending down into their determinants, and recalling that in the flexible-price model output Y is always equal to potential output Y*:

$$S_p = \left(1 - C_y\right)(1 - t)Y^* - C_0$$

Find the interest rate r that makes investment spending equal to the sum of the three components of the flow of saving into financial markets. Then use that interest rate to calculate whatever economic variables of interest you seek.

This procedure will work 99% of the time in solving problems posed and analyzing situations described by the flexible-price model.

Money and Inflation (Under Flexible Prices)

The key equation to use in solving money-and-inflation problems (under flexible prices) is the quantity theory of money as expressed in the quantity equation:

$$M \times V = P \times Y$$

where M is the *money stock*—the amount of wealth that the households and businesses of the economy hold in liquid, readily-spendable form to finance their purchases and to serve as a buffer against unexpected changes in their incomes, their purchases, or demands for repayment—V is the so-called *velocity* of money, P is the aggregate price level, and Y is the level of real GDP. Note that from one perspective the quantity theory of money is an *identity*: velocity V is *defined* to make the equation above true.

The quantity equation changes from being an identity true by definition to a theory useful for analyzing the economy if one imposes another equation for velocity, almost always either a declaration that velocity is growing over time at a constant proportional rate v:

$$V_{t+1} = V_t \times (1+v)$$

or that velocity depends on the nominal interest rate i:

$$V = V_0 + V_i \times i$$

and that the nominal interest rate i is the sum of the inflation rate π and the (usually constant in money-and-inflation problems) real interest rate r:

$$i = r + \pi$$

The canonical money-and-inflation problem will then take one of two forms. First, the problem might give one the information to determine three of M, V, P, and Y, and ask one to calculate the fourth. Second, the problem might give one the information to determine the rates-of-change of three of M, V, P, and Y, and ask one to calculate the rate-of-change of the fourth. If asked in level form (and if velocity V is not the unknown to be calculated, nine times out of ten the solution will become transparent if one expands the determinants of velocity in the quantity equation thus, either:

$$M_t \times \left(V_0 \times (1+v)^t\right) = P_t \times Y_t$$

or:

$$M \times \left(V_0 + V_i \times (r + \pi)\right) = P \times Y$$

Remember, the information you need to calculate the velocity of money may not be immediately obvious. It may be hidden in an aside that allows you to calculate the time trend of velocity, or the nominal interest rate and the dependence of velocity upon the nominal interest rate.

If asked in rate-of-change form, remember that the quantity equation implies that:

$$m + v = p + y$$

where m, v, p, and y are the proportional growth rates of the money stock, the velocity of money, the price level, and output respectively. Once you write down this rate-of-change form of the quantity equation, the answer to a rate-of-change problem will almost always be obvious.

Income and Expenditure Under Sticky Prices

The fourth of the sets of problems from which the exam questions might be drawn are problems based on the Keynesian sticky-price income-expenditure model. It is easy to solve such problems quickly as long as you maintain your grasp on three concepts:

- Equilibrium is a stable level of inventories: production and income Y equal to expenditure E.
- The multiplier μ.
- The level of autonomous spending A.

In the sticky-price model, the equilibrium condition is that inventories be stable. If inventories are rising, businesses will cut back production, fire workers, and production and incomes will fall. If inventories are falling, businesses will raise production, hire workers, and production and inventories will rise. Only when production and income Y are equal to expenditure E—with E given by:

$$E = C + I + G + NX$$

will a sticky-price economy be in equilibrium.

The second important concept to grasp is the multiplier. In the sticky-price income-expenditure model, a change in autonomous spending will have a larger than one-for-one effect on the equilibrium level of output because a fall in production will reduce incomes, which will reduce demand, which will induce a further fall in production which will in its turn reduce incomes still further. The value of this multiplier μ is given by:

$$\mu = \frac{1}{1 - C_y(1-t) + IM_y}$$

where:

- C_y is the marginal propensity to consume
- t is the tax rate
- IM_y is the share of incomes spent on imported goods and services.

The third and last important concept to grasp is autonomous spending A, for calculating the equilibrium level of income and output Y is trivial once one knows autonomous spending A and the multiplier m. It is:

$$Y = \mu \times A$$

with autonomous spending A equal to the sum of:

- Baseline consumption spending C_0
- Investment spending $I = I_0 + I_r$ x r, with r being the real interest rate

- Government spending G
- Gross exports $GX = X_f Y^f + X_\varepsilon \varepsilon = X_f Y^f + X_\varepsilon(\varepsilon_0 - \varepsilon_r(r - r^f))$, with Y^f being foreign incomes, ε being the real exchange rate, ε_0 being foreign exchange speculators' confidence in the currency, r being the domestic real interest rate, and r^f being the foreign real interest rate.

Sticky Price Interest Rates, Production, and Equilibrium Questions

The fifth of the sets of problems from which the exam questions might be drawn are problems based on the Keynesian IS-Curve model that deal with the interest rate, the level of production, and equilibrium. The key concepts for this model are (a) the Federal Reserve chooses a real interest rate in an attempt to keep the economy near full employment—to avoid accelerating inflation on the one hand and deep depression on the other—(b) the IS Curve, a function that tells us the relationship between the real interest rate r and equilibrium real GDP Y. The IS Curve is given by:

$$Y = \mu A_0 + \mu\left(I_r + X_\varepsilon \varepsilon_r\right)r$$

with:

- r being the real interest rate
- μ being the multiplier, $\mu = 1/(1-C_y(1-t)+IM_y)$
- I_r being the sensitivity of investment to the real interest rate r
- X_ε x ε_r being the sensitivity of gross exports to the real interest rate r

And (c) baseline autonomous spending, with baseline autonomous spending A_0 equal to the sum of:

- Baseline consumption spending C_0
- Baseline investment spending I_0
- Government spending G
- The dependence-of-exports-on-foreign-income term $X_f Y^f$
- The dependence-of-exports-on-speculators'-expectations term $X_\varepsilon \varepsilon_0$
- The dependence-of-exports-on-foreign-interest-rates term $X_\varepsilon \varepsilon_r r^f$

These three concepts will not, by themselves, allow you to solve all interest rate, production, and equilibrium problems, for there are many additional wrinkles—balance of payments constraints, LM curves, credit crunches, and so forth that are often added to the model. But your chances of solving such problems are zero unless you have a firm grasp of these three concepts: the Federal Reserve's power to choose the interest rate r, the IS curve, and the definition of baseline autonomous spending A_0.

Inflation and Expectations Questions

The key concept is the Phillips curve:

$$\pi = \pi^e - \beta \times (u - u^*) + \varepsilon^s$$

where π is the rate of inflation, π^e is the expected rate of inflation, $-\beta$ x (u-u*) is the effect on inflation of a boom (unemployment u below the natural rate of unemployment u*) or a recession (unemployment u above the natural rate of unemployment u*), and ε^s representing supply shocks—like the 1973 oil price increase—that can directly affect the rate of inflation.

You will solve inflation-and-expectations questions in five conceptual steps. First, use the IS curve model, your knowledge of how the Federal Reserve is setting the interest rate, and Okun's law that relates the level of GDP to the unemployment rate to calculate the unemployment rate. Second, determine the natural rate of unemployment u*. Third, determine the rate of expected inflation π^e. Fourth, use all these pieces of information to calculate actual inflation π. Fifth, repeat if necessary if the problem asks for answers not just for the current year but for a number of future years.

There are two likely complications. The first is that there may well be some simultaneity here: it may well be the case that Federal Reserve policy depends on the expected future inflation rate which itself depends on the current interest rate. The second is that inflation expectations may be set in any of three ways. The economy may have:

1. Static expectations, in which people assume that inflation will be low, irrelevant, and negligible and do not change their minds.
2. Adaptive expectations, when people assume that inflation next year will be like inflation was this year.
3. Rational expectations, where people use all of the information they have to forecast inflation, and thus where (if there are no big surprises) actual inflation π is equal to expected inflation π^e.

If inflation expectations are *static,* people just don't think about inflation. There will be some years in which unemployment is relatively low; and in those years inflation will be relatively high. There will be other years in which unemployment is higher, and then inflation will be lower. But the trade-off between inflation and unemployment will not change from year to year.

As long as inflation last year is a good guide to inflation this year, workers, investors, and managers are likely to hold *adaptive* expectations. Under *adaptive expectations,* the Phillips curve will shift up or down depending on whether last year's inflation was higher or lower than the previous year's. Inflation accelerates when unemployment is less than the natural unemployment rate, and decelerates when unemployment is more than the natural rate. Hence this Phillips curve is sometimes called the accelerationist Phillips curve.

If government policy and the economic environment are changing rapidly enough that adaptive expectations lead to significant errors, the economy will shift to “rational” expectations. Under rational expectations, people forecast inflation by looking at what current and expected future government policies tell us about what inflation will be. Under rational expectations, anticipated changes in economic policy turn out to have no effect on the level of production or employment.

To see this, let's consider an economy in which workers, managers, savers, and investors have rational expectations, let's suppose that the central bank reacts to higher inflation by raising real interest rates, and let's suppose the government increases government spending to try to reduce unemployment. What is likely to happen? People who understand that government spending is about to increase will raise their expectations of inflation. Higher expected inflation will raise actual inflation too. And so the central bank will react by raising interest rates, offsetting the stimulative effect of the additional government spending. The likely result is an increase in inflation, but no increase in real GDP or reduction in unemployment.