Chapter 4: Growth Theory

(Alternative Draft)[1]

J. Bradford DeLong
delong@econ.berkeley.edu
http://www.macro-online.org/

November 2001

Questions

1. What are the principal determinants of long-run economic growth?
2. What equilibrium condition is useful in analyzing long-run growth?
3. How quickly does an economy head for its balanced-growth path?
4. What effect does faster population growth have on long-run growth?
5. What effect does a higher savings rate have on long-run growth?

---

[1] An alternative draft of chapter 4 of my textbook, *Macroeconomics* (Burr Ridge, IL: McGraw-Hill, 2001). This draft is a little bit simpler and thus—I hope—more approachable than the draft in the textbook. Of course, it covers less. I oscillate back and forth with respect to which of the versions I prefer.

4.1 Background: Sources of Growth

Ultimately long-run growth is *the* most important aspect of how the economy performs. Material standards of living and levels of economic productivity in the United States today are about four times what they are today in, say, Mexico—and five or so times what they were at the end of the nineteenth century—because of rapid, sustained long-run economic growth. Good and bad policies can accelerate or cripple this growth. Argentines were richer than Swedes before World War I, but Swedes today have four times the standard of living and the productivity level of Argentines. Almost all of this difference is due to differences in growth policies working through two channels. The first is the impact of policies on the economy's *technology* that multiplies the efficiency of labor. The second is their impact on the economy's *capital intensity*—the stock of machines, equipment, and buildings.

In this growth section of the textbook the following chapter, Chapter 5, analyzes the facts of economic growth. This chapter, Chapter 4, focuses on the theory of economic growth. Its aim is to build up the growth model that economists use to analyze how much growth is generated by the advance of technology and how much by investment to boost capital intensity on the other.

Better Technology

The bulk of the reason that Americans today are vastly richer and more productive than their predecessors of a century ago is better technology. We now know how to make electric motors, dope semiconductors, transmit signals over fiber optics, fly jet airplanes, machine internal combustion engines, build tall and durable structures out of concrete and steel, record entertainment programs on magnetic tape, make hybrid seeds, fertilize crops with nutrients, organize assembly lines, and a host of other things our predecessors did not know how to do. Better technology leads to a higher *efficiency of labor*--the skills and education of the labor force, the ability of the labor force to handle modern machine technologies, and the efficiency with which the economy's businesses and markets function.

Capital Intensity

However, a large part is also played by the second factor: *capital intensity*. The more capital the average worker has at his or her disposal to amplify productivity, the more prosperous the economy will be. In turn, there are two principal determinants of capital intensity. The first is the *investment effort* made by the economy: the share of total production--real GDP-- saved and invested to boost the capital stock. The second are the economy’s *investment requirements*: how much new investment is needed to simply equip new workers with the standard level of capital, to keep up with new technology, and to replace worn- machines and buildings.

The ratio between the investment effort and the investment requirements of the economy determines the economy's capital intensity. Capital intensity is measured by the economy’s capital-output ratio K/Y—the economy’s capital stock K divided by its annual real GDP Y—which we will write using a lower-case Greek kappa, κ.

$$\kappa = \frac{K}{Y}$$

Recap 4.1: Sources of Long Run Growth

Ultimately, long-run economic growth is *the* most important aspect of how the economy performs. Two major factors determine the prosperity and growth of an economy: the pace of technological advance and the capital intensity of the economy. Policies that accelerate innovation or that boost investment to raise capital intensity accelerate economic growth.

4.2 The Balanced-Growth Path

In economists' standard *growth model*[2] the type of equilibrium they study is a *balanced-growth equilibriu*m. In the balanced-growth equilibrium the capital intensity of the economy—its capital stock divided by its total output—is constant. However, other variables like the capital stock, real GDP, and output per worker are growing.

Economists use the standard model to calculate the balanced-growth path. They then forecast that if the economy is on this path, it will grow along this path. And they forecast that if the economy is not on its balanced growth path, it will head toward that path.

The Steady-State Capital-Output Ratio

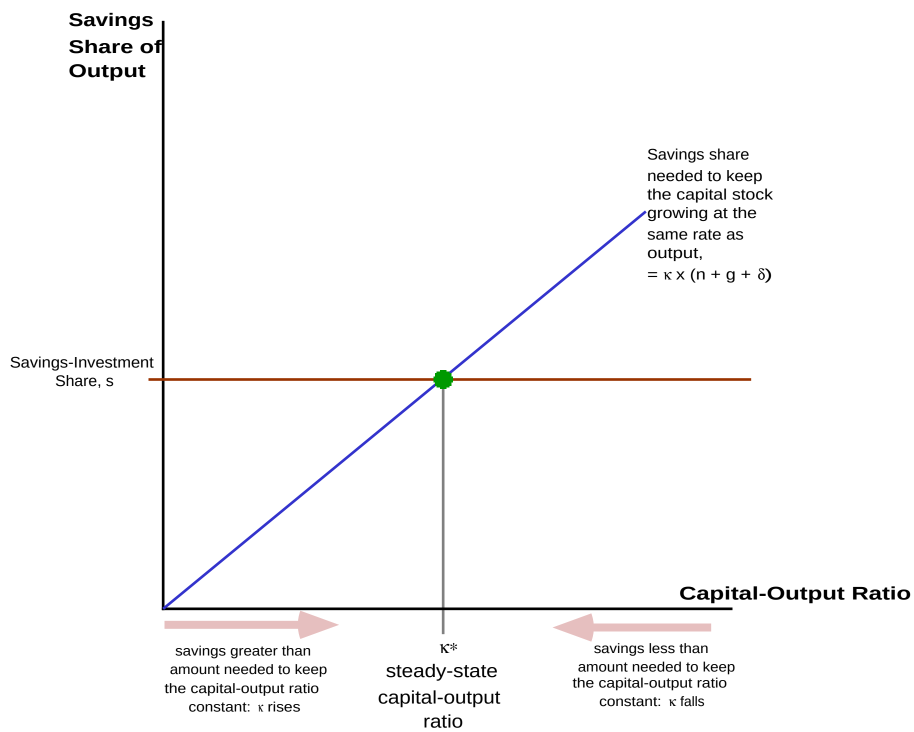
What is the economy's balanced-growth path? On the balanced-growth path, the economy's capital-output ratio—which as you recall we write with a Greek letter kappa thus: κ—is equal to a particular steady-state value, which we will call κ^*. (The "*" is often used in economics to denote a particular value of a variable for which the economy is in some kind of equilibrium, to which the economy tends to converge, or around which the economy tends to fluctuate.) We calculate this steady-state value of the capital-output ratio κ^* by taking the share of

---

[2] The standard model is called the Solow model, after Nobel Prize-winning M.I.T. economist Robert Solow.

production that is saved and invested for the future—the economy's saving-investment rate s—and then dividing it by the sum of the depreciation rate at which capital wears out (written δ), the proportional growth rate (written n) of the labor force, and the proportional growth rate (written g) of the efficiency of labor.[3]

Figure 4.1: Why κ* Is the Equilibrium Capital-Output Ratio



If the current capital-output ratio is equal to its steady-state value κ*, then the share of output saved and invested every year is exactly what is needed to keep the capital stock growing at the same rate as output, and keep the capital-output ratio constant.

In algebra:

[3] Recall that we call these last three *investment requirements*.

$$\kappa^* = \frac{s}{n + g + \delta}$$

Along the balanced-growth path, the level of output per worker Y/L is found by raising the steady-state capital-output ratio k* to the power of the growth multiplier (written λ)[4], and then multiplying the result by the current efficiency of labor (written E_t). In algebra:

$$\frac{Y_t}{L_t} = \kappa^{*\lambda} \times E_t$$

The steady-state capital-output ratio κ* is constant (as long as the economy's savings-investment share s, its labor force growth rate n, and its efficiency of labor growth rate g do not change). However, the balanced-growth path level of output per worker is not constant. As time passes, the balanced-growth path level of output per worker rises. Why? Because output per worker Y/L is equal to the *current* efficiency of labor E_t times the steady-state capital-output ratio κ* raised to the power λ; and technological progress means that the efficiency of labor E_t grows at a proportional growth rate g.

Is the economy always on its balanced-growth path? No. But if the economy is not on it, it is heading towards it.

---

[4] λ, the growth multiplier, is

$$\lambda = \frac{\alpha}{1 - \alpha}$$

where α is the diminishing-returns-to-scale parameter from last chapter's production function $Y/L = (K/L)^{\alpha} \times E^{1-\alpha}$. It tells by how much (in percentage terms) the economy's output would rise if its capital stock were to grow by one percent.

Box 4.1: Details: The Determinants of the Balanced-Growth Path

Thus the steady-state balanced growth path depends on five factors:

- the economy's savings-investment rate, the share of output used to buy investment goods to boost the capital stock (written s)
- the growth rate of the efficiency of labor (written g)
- the depreciation rate—the proportion of the existing capital stock K that wears out or becomes obsolete every year (written δ)
- the economy's labor force growth rate (written n)
- the economy's growth multiplier (written λ, equal to $\alpha/(1-\alpha)$, where α comes from the production function)
- the current efficiency of labor—a measure of the economy's ability to use technology, where "technology" is defined in the broadest possible sense to include work organization, incentives, and all other factors that affect the ability of the economy to use resources to produce goods and services. (written E_t).

Factors (1) through (4) determine the steady-state capital-output ratio κ^*. which is then raised to the λ power (factor (5)), and the result is then multiplied by the current efficiency of labor E_t (factor (6)).

If the capital-output ratio κ is below κ^*, the share of output invested each year (equal to s) generates a greater volume of investment than is needed to keep the capital stock growing as fast as output. Capital and output would be growing at the same proportional rate—and the capital-output ratio would be constant—if the share of output saved and invested were equal to $\kappa(n + g + \delta)$).

Thus as long as κ is less than κ^*, the capital-output ratio is rising. Moreover, if the capital-output ratio is above κ^*, the share of national product saved and invested each year (equal to s) is less than the share needed to keep the capital stock growing as fast as output (which is still equal to $\kappa(n + g + \delta)$). The capital-output ratio is falling. Thus either way—whether κ is above or below κ^*--the capital-output ratio κ changes to close some of the gap between its current value and its steady-state equilibrium value κ^*.

Forecasting the Economy's Destiny

The standard Solow growth model makes forecasting an economy's long-run growth destiny simple:

1. Calculate the steady-state capital-output ratio, $\kappa^*=s/(n+g+\delta)$, equal to the savings share divided by the investment requirements.
2. Amplify the steady-state capital-output ratio κ^* by raising it to the power of the growth multiplier $\lambda = (\alpha/(1-\alpha))$, where α is the production function's diminishing-returns-to-scale parameter.
3. Multiply the result by the current efficiency of labor E_t.

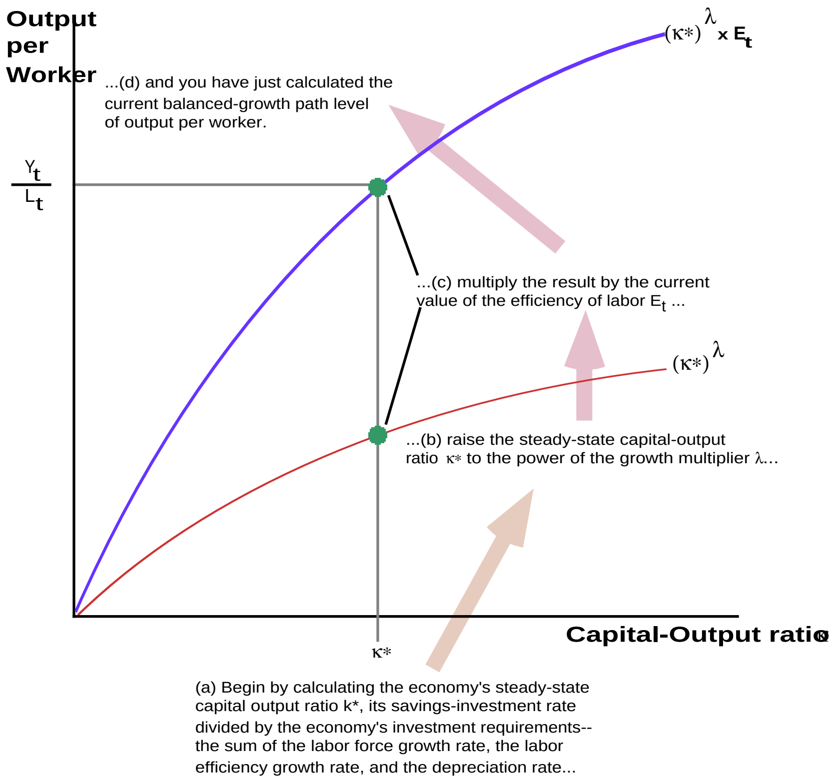
You have just calculated output per worker on the economy's *balanced-growth path*. If you just want to understand the present, you are done. If you want to also forecast the future, then:

4. Forecast that balanced-growth output per worker will grow at the same proportional rate g as labor efficiency.

If the economy is on its balanced-growth path, you are done. But if the economy is not currently on its balanced-growth path, then:

5. Forecast that the economy is heading for its balanced-growth path.
6. Forecast that the economy will grow along its balanced-growth path after it has converged to it.

Figure 4.2 How to Calculate the Economy's Balanced-Growth Path



Calculating the balanced-growth level of output per worker is simple: (i) calculate the steady-state capital-output ratio κ*, (ii) raise κ* to the power of the growth multiplier λ, and (iii) multiply the result by the efficiency of labor E_t.

The growth model makes forecasts of the long-run destiny of the economy straightforward, and provides an easy way to analyze how the factors making for (a) higher capital intensity and (b) better technology and labor efficiency determine output per worker.

Why, and how, does this growth model work? Why is there a steady-state growth path? Why do these calculations above tell us what it is?

To understand these issues, we need to back up and dig a little deeper. To explain them is the business of the rest of Chapter 4.

Recap 4.2: The Balanced-Growth Path

The standard growth model focuses on four key concepts: the level of output per worker, the steady-state capital-output ratio (determined by the balance between the share of total output saved and invested and the investment requirements—the sum of the labor force growth, labor efficiency growth, and depreciation rates—of the economy), the growth multiplier (determined by the extent of diminishing returns in the production function), and the efficiency of labor (which grows as technology progresses). In balanced-growth equilibrium, the first of these—output per worker—is equal to the steady-state capital-output ratio raised to the power of the growth multiplier, times the current level of the efficiency of labor.

4.3 The Standard Growth Model

Economists begin to analyze long-run growth as they begin to analyze any situation: by building a simple, standard model, the Solow model. Economists then look for an *equilibrium* of the model—a point of balance, a condition of rest, a state of the system toward which the model will converge over time. Once you have found the equilibrium position toward which the economy tends to move, you can use it to understand how the model will behave. If you have built the right model, this will tell you in broad strokes how the economy will behave.

In economic growth economists look for the *balanced-growth equilibrium*. In the balanced-growth equilibrium the capital intensity of the economy is stable. The economy's capital stock and its level of real GDP are growing at the same proportional rate. And the capital-output ratio--the ratio of the economy's capital stock to annual real GDP--is constant.

The Production Function

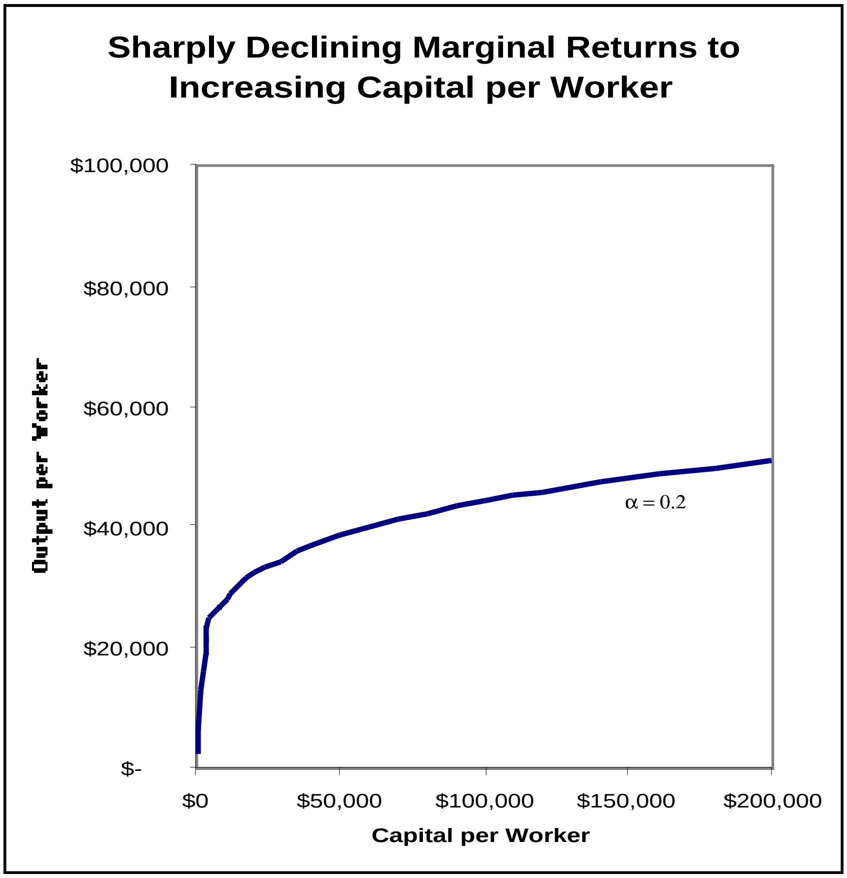
The first component of the model is a *behavioral relationship* called the *production function.* This behavioral relationship tells us how the productive resources of the economy—the labor force, the capital stock, and the level of technology that determines the efficiency of labor—can be used to produce and determine the level of output in the economy. The total volume of production of the goods and services that consumers, investing businesses, and the government wish for is limited by the available resources. The production function tells us how available resources limit production.

Tell the production function what resources the economy has available, and it will tell you how much the economy can produce. Abstractly, we write the production function as:

$$(Y/L) = F((K/L), E)$$

This says that real GDP per worker (Y/L)--real GDP Y divided by the number of workers L—is systematically related, in a pattern prescribed by the form of the function F(), to the economy's available resources: the capital stock per worker (K/L), and the current efficiency of labor (E) determined by the current level of technology and the efficiency of business and market organization.

Figure 4.3: The Cobb-Douglas Production Function, for Parameter α Near Zero



When the parameter α is close to zero, an increase in capital per worker produces much less in increased output than the last increase in capital per worker. Diminishing returns to capital accumulation set in rapidly and ferociously.

The Cobb-Douglas Production Function

As long as the production function is kept at the abstract level of an F()—one capital letter and two parentheses—it is not of much use. We know that there is a relationship between resources and production, but we don't know it is. To make things less abstract—and more useful--hwe will use one particular form of the production function. We will use the so-called Cobb-Douglas production function because it makes many kinds of calculations relatively simple.

The Cobb-Douglas production function states that:

$$Y = K^{\alpha} \times (E \times L)^{1-\alpha}$$

The economy's level of output Yis equal to the capital stock raised to the exponential power of some number α, multiplied by the product of the labor force L and the current efficiency of labor E, themselves raised to the exponential power (1- α).

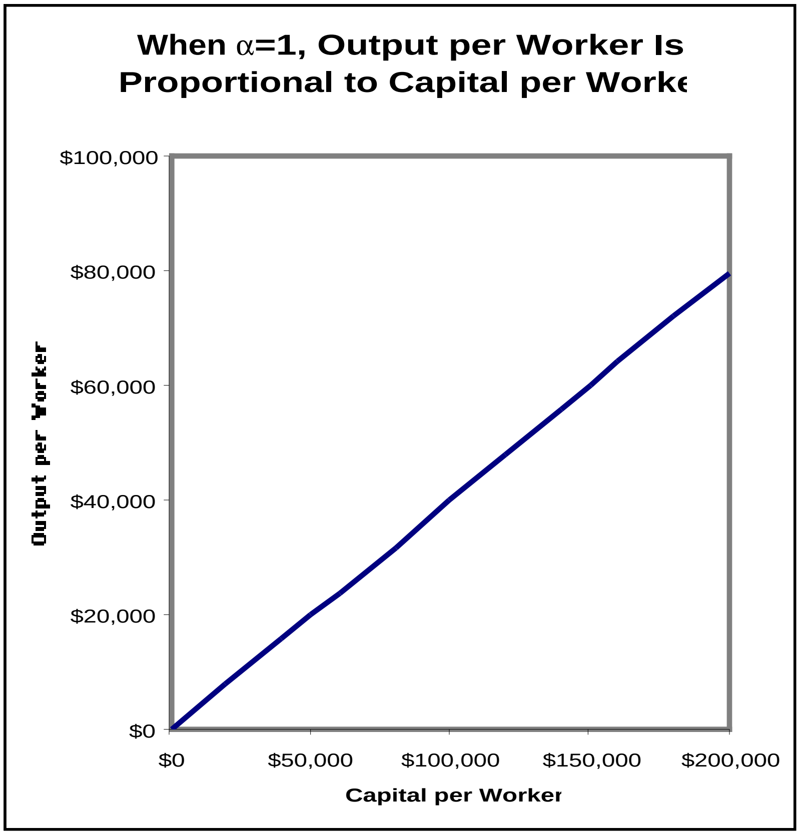
Alternatively, in the output per worker form that we can derive by dividing both sides of the equation by the labor force L, we can write this production function as:

$$(Y/L) = (K/L)^{\alpha} \times (E)^{1-\alpha}$$

Output per worker (Y/L) is equal to the capital stock per worker K/L raised to the exponential power of some number α, and then multiplied by the current efficiency of labor E raised to the exponential power (1- α). Both forms of the production function are useful.

The efficiency of labor E and the number α are *parameters* of the model. The parameter α is always a number between zero and one. The best way to think of it as the parameter that governs how fast diminishing returns to investment set in. A level of α near zero means that the extra output made possible by each additional unit of capital declines very quickly as capital increases, as Figure 4.3 shows.

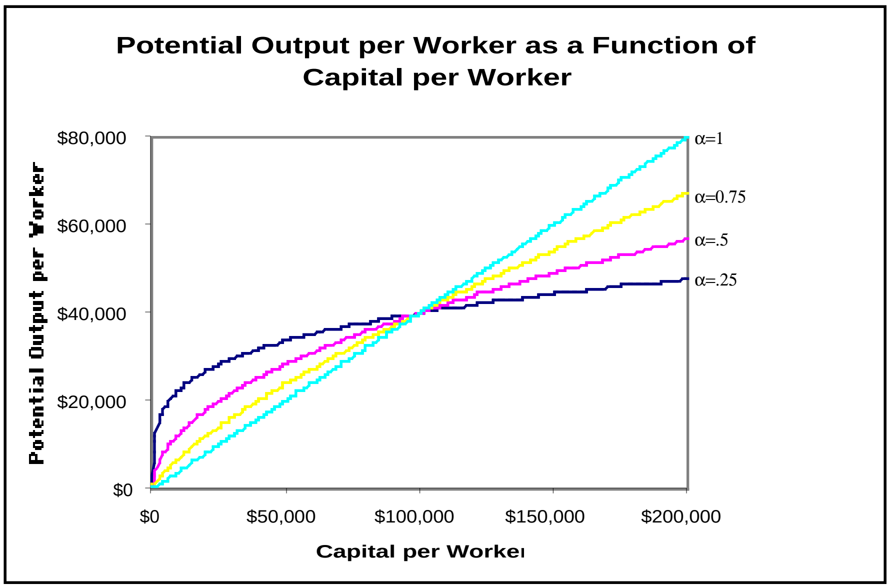
Figure 4.4: The Cobb-Douglas Production Function, for Parameter α Near One



When α=1, doubling capital per worker doubles output per worker. There are no diminishing returns to capital accumulation. When the parameter α is near to but less than one, diminishing returns to capital accumulation set in slowly and gently.

By contrast, a level of α near one means that the next additional unit of capital makes possible almost as large an increase in output as the last additional unit of capital, as Figure 4.4 shows. When α equals one, output is proportional to capital: double the capital stock, and you double output as well. When α is near to but less than one, diminishing returns to capital accumulation do set in, but they do not set in rapidly or steeply.

Figure 4.5: The Cobb-Douglas Production Function Is Flexible



By changing α--the exponent of the capital-labor ratio (K/L) in the Cobb-Douglas production function--you change its curvature, and thus how fast diminishing returns to further increases in capital per worker set in. Raising the parameter α increases the speed with which the returns to increased capital accumulation diminish. Thus we call α the diminishing returns to scale parameter.

The other parameter E tells us the current level of the efficiency of labor. A higher level of E means that more output per worker can be produced for each possible value of the capital stock per worker. A lower value of E means that the economy is very unproductive: not even huge amounts of capital per worker will boost output per worker to achieve what we would think of as prosperity. Box 4.2 illustrates how to use the production function once you know its form and parameters--how to calculate output per worker once you know the capital stock per worker.

The Cobb-Douglas production function is "flexible" in the sense that it can be tuned to fit any of a wide variety of different economic situations. Figure 4.5 shows a small part of the flexibility of the Cobb-Douglas production function. Is the level of productivity high? The Cobb-Douglas function can fit with a high initial level of the efficiency of labor E. Does the economy rapidly hit a wall as capital accumulation proceeds and find that all the investment in the world is doing little to raise the level of production? Then the Cobb-Douglas function can fit with a low level--near zero--of the diminishing-retuns-to-capital parameter α. Is the speed with which diminishing-returns-to-investment sets in moderate? Then pick a moderate value of α, and the Cobb-Douglas function will once again fit.

No economist believes that there is, buried somewhere in the earth, a big machine that forces the level of output per worker to behave exactly as calculated by the algebraic production function above. Instead, economists think that the Cobb-Douglas production function above is a simple and useful approximation.

The true process that does determine the level of output per worker is an immensely complicated one: everyone in the economy is part of it. And it is too complicated to work with. Writing down the Cobb-Douglas production function is a breathtakingly large leap of abstraction. Yet it is a useful leap, for this approximation is good enough that using it to analyze the economy will get us to approximately correct conclusions.

Box 4.2: An Example: Using the Production Function

For given values of E (say 10000) and α (say 0.3), this production function tells us how the capital stock per worker is related to output per worker. If the capital stock per worker were $250,000, then output per worker would be:

$$Y/L = (\$250000)^{0.3} \times (10000)^{0.7}$$
$$Y/L = \$41.628 \times 630.958$$
$$Y/L = \$26,265$$

And if the capital stock per worker were $125,000, then output per worker would be:

$$Y/L = (\$125000)^{0.3} \times (10000)^{0.7}$$
$$Y/L = \$33.812 \times 630.958$$
$$Y/L = \$21,334$$

Note that the first $125,000 of capital boosted production from $0 to $21,334, and that the second $125,00 of capital boosted production from $21,334 to $26,265: less than a quarter as much. These substantial diminishing returns should not be a surprise: the value of α in this example--0.3--is low, and low values are supposed to produce rapidly diminishing returns to capital accumulation.

Now nobody expects anyone to raise $250,000 to the 0.3 power in their head and come up with 41.628. That is what calculators are for. This Cobb-Douglas form of the production function with its fractional exponents carries the drawback that we cannot expect students (or professors!) to do problems in their heads or with just pencil-and-paper. However, this Cobb-Douglas form of the production function also carries substantial benefits: by varying just two numbers--the efficiency of labor E and the diminishing-returns-to-capital parameter α--we can consider and analyze a very broad set of relationships between resources and the economy's productive power.

In fact, this particular Cobb-Douglas form for the production function with all these αs and (1-α)s as exponents was built by Cobb and Douglas for precisely for this purpose: so that it would be simple to, by judicious choice of different values of E and α, "tune" the function so that it could capture a large range of different kinds of behavior.

The Rest of the Growth Model

The rest of the growth model is straightforward. First comes the need to keep track of the quantities of the model over time. Do so by attaching to each variable--like the capital stock or the efficiency of labor or output per worker or the labor force--a little subscript telling what year it applies to. Thus K_{1999} will be the capital stock in year 1999. If we want to refer to the efficiency of labor in the current year (but don't much care what the current year is), we will use a *t* (for "time) as a placeholder to stand in for the numerical value of the current year. Thus we write: E_t. And if we want to refer to the efficiency of labor in the year after the current year, we will write: E_{t+1}.

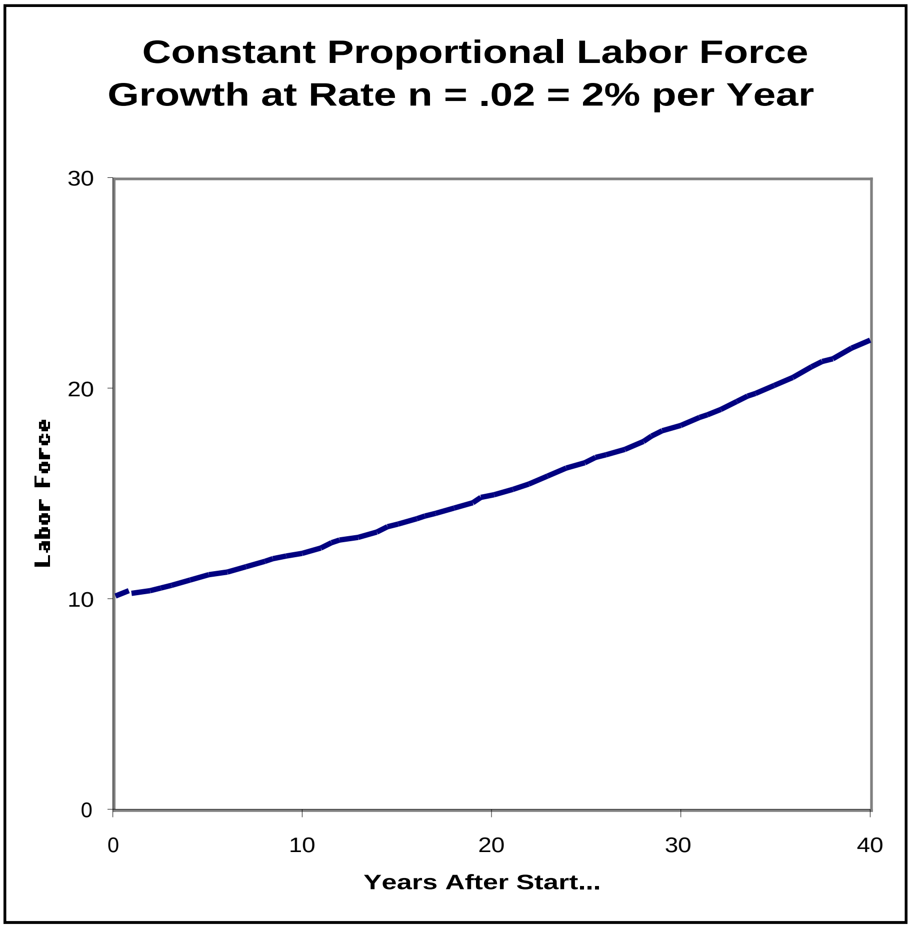
Population Growth

Second comes the pattern of labor force growth. We assume—once again making a simplifying leap of abstraction--that the labor force L of the economy is growing at a constant proportional rate given by the value of a parameter *n*. Note that n does not have to be the same across countries, and can shift over time in any one country). Thus between this year and the next the labor force will grow so that:

$$L_{t+1} = (1+n) \times L_t$$

Next year's labor force will be n percent higher than this year's labor force, as Figure 4.6 shows.

Figure 4.6: Constant Labor Force Growth



A labor force increasing at a rate of 2% per year will double roughly every 35 years.

Thus if this year's labor force were 10 million, and the growth rate parameter n were 2 percent per year, then next year's labor force would be:

$$L_{t+1} = (1+n) \times L_t$$
$$L_{t+1} = (1+2\%) \times L_t$$
$$L_{t+1} = (1+0.02) \times 10$$
$$L_{t+1} = 10.2 \ \ million$$

We assume that the rate of growth of the labor force is constant not because we believe that labor force growth is unchanging, but because it makes the analysis of the model simpler. This tradeoff between realism in the model's description of the world and simplicity as a way to make the model easier to analyze is one that economists always face. They usually resolve it in favor of simplicity.

Efficiency of Labor

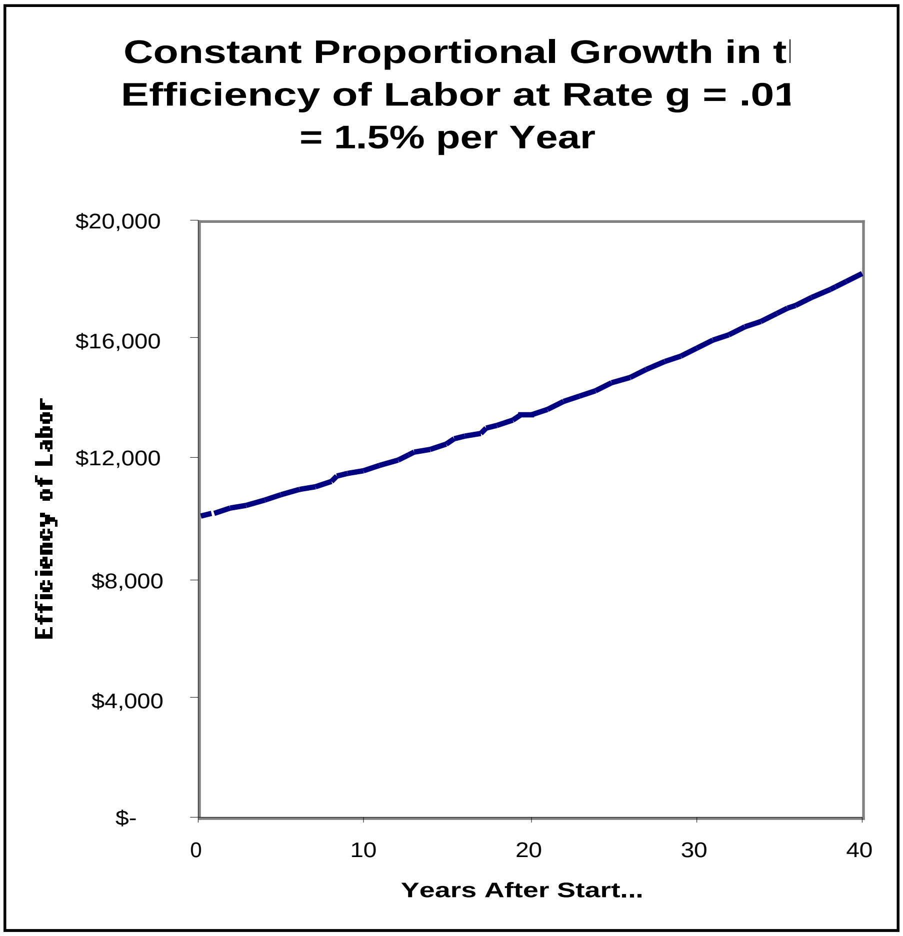
Assume, also, that the efficiency of labor E is growing at a constant proportional rate given by a parameter g. (Note that g does not have to be the same across countries, and can shift over time in any one country.) Thus between this year and the next year:

$$E_{t+1} = (1+g) \times E_t$$

Next year's level of the efficiency of labor will be g percent higher than this year's level, as Figure 5 shows. Thus if this year's efficiency of labor were \$10,000 per year, and the growth rate parameter g were 1.5 percent per year, then next year the efficiency of labor would be:

$$E_{t+1} = (1+g) \times E_t$$
$$E_{t+1} = (1+0.015) \times \$10,000$$
$$E_{t+1} = \$10,150$$

Figure 4.7: Constant Growth in the Efficiency of Labor



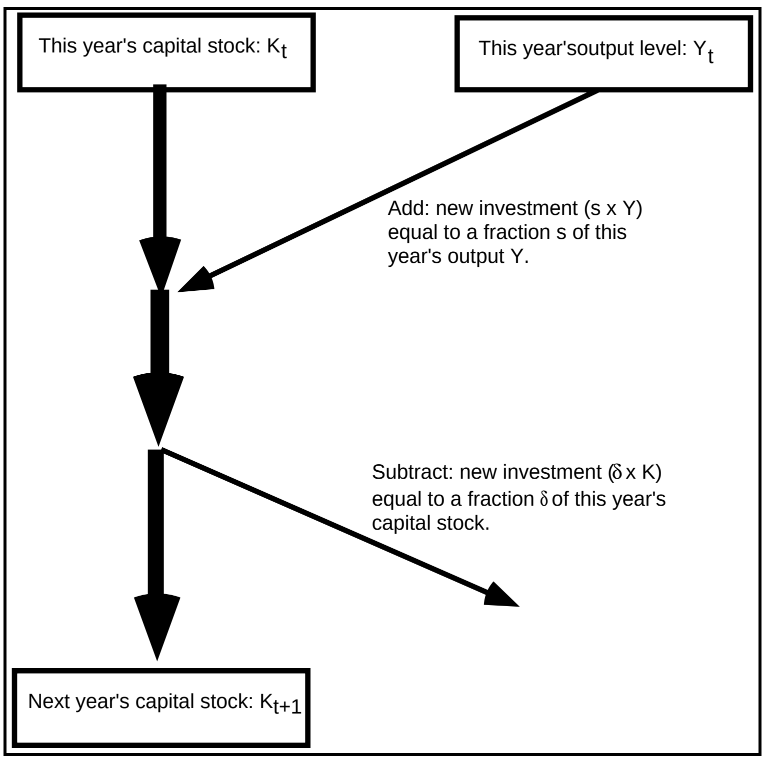
If the efficiency of labor grows at a constant proportional rate of 1.5 percent per year, it will take about 47 years for it to double.

Once again this assumption is made because it makes the analysis of the model easier, not because the rate at which the efficiency of labor grows is constant.

Savings and Investment

Last, assume that a constant proportional share, equal to a parameter s, of real GDP is saved each year and invested. These gross investments add to the capital stock, so a higher amount of savings and investment means faster growth for the capital stock.

Figure 4.8: Changes in the Capital Stock



Gross savings and investment adds to and depreciation subtracts from the capital stock. Depreciation is a share δ of the current capital stock. Savings-investment is a share s of current production.

But the capital stock does not grow by the full amount of *gross* investment. A fraction δ (the Greek letter lower-case delta, for depreciation) of the capital stock wears out or is scrapped each period. Thus the actual relationship between the capital stock now and the capital stock next year is:

$$K_{t+1} = K_t + (s \times Y_t) - (\delta \times K_t)$$

The level of the capital stock next year will be equal to the capital stock this year, plus the savings rate s times this year's level of real GDP, minus the depreciation rate δ times this year's capital stock, as Figure 4.6 shows. Box 4.2 illustrates how to use this capital accumulation equation to calculate the capital stock.

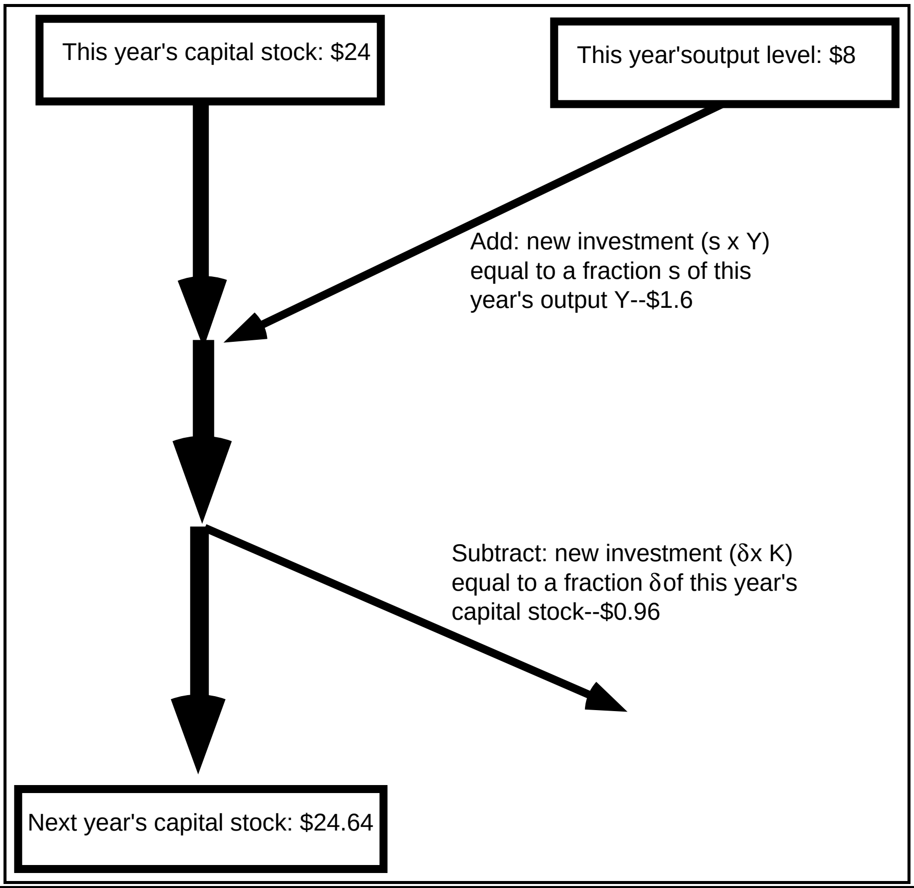
Box 4.3: An Example: Investment, Depreciation, and Capital Accumulation

For example, suppose that the current level of output in the economy is $8 trillion a year and the current year's capital stock in the economy is $24 trillion. Then a savings rate s of 20 percent and an annual depreciation rate δ of 4 percent would mean that next year's capital stock will be:

$$K_{t+1} = K_t + (s \times Y_t) - (\delta \times K_t)$$
$$K_{t+1} = \$24 + (0.2 \times \$8) - (0.04 \times \$24)$$
$$K_{t+1} = \$24 + \$1.6 - \$0.96$$
$$K_{t+1} = \$24.64 \quad trillion$$

Between this year and next year the capital stock has grown by $640 billion. That is a proportional growth rate of 2.667%.

Figure 4.9: Additions to and Subtractions from the Capital Stock



That is all there is to the growth model: three assumptions about rates of population growth, increases in the efficiency of labor, and investment, plus one additional equation to describe how the capital stock grows over time. Those plus the production function make up the growth model. It is simple. But understanding the processes of economic growth that the model generates is more complicated.

Recap 4.3: The Standard Growth Model

When the economy's capital stock and its level of real GDP are growing at the same proportional rate, its capital-output ratio--the ratio of the economy's capital stock to annual real GDP--is constant, and the economy is in equilibrium--on its steady-state balanced growth path. The standard growth model an analyzes how this steady-state balanced growth path is determined by five factors: the level of the efficiency of labor, the growth rate of the efficiency of labor, the economy's savings rate, the economy's population growth rate, and the capital stock depreciation rate.

4.4 Understanding the Growth Model

Economists' first instinct when analyzing any model is to look for a point of *equilibrium.* They look for a situation in which the quantities and prices being analyzed are stable and unchanging. And they look for the economic forces to push an out-of-equilibrium economy to one of its points of equilibrium. Thus microeconomists talk about the equilibrium of a particular market. Macroeconomists talk (and we will talk later on in the book) about the equilibrium value of real GDP relative to potential output.

In the study of long-run growth, however, the key economic quantities are never stable. They are growing over time. The efficiency of labor is growing, the level of output per worker is growing, the capital stock is growing, the labor force is growing. How, then, can we talk about a point of equilibrium where things are stable if everything is growing?

The answer is to look for an equilibrium in which everything is growing together, at the same proportional rate. Such an equilibrium is one of *steady-state balanced growth.* If everything is growing together, then the relationships between key quantities in the economy are stable. And it makes this chapter easier if we focus on one key ratio: the capital-output ratio. Thus our point of equilibrium will be one in which the capital-output ratio is constant over time, and toward which the capital-output ratio will converge if it should find itself out of equilibrium.

How Fast Is the Economy Growing?

So how fast are the key quantities in the economy growing? We know that they are growing. The efficiency of labor is, after all, increasing at the proportional rate g. The labor force is increasing at the proportional rate of growth n. It is easy to understand how fast the quantities in the economy are growing is straightforward if we remember our three mathematical rules:

1. The proportional growth rate of a product --P x Q, say--is equal to the sum of the proportional growth rates of the factors, is equal to the growth rate of P plus the growth rate of Q.

2. The proportional growth rate of a quotient--E/Q, say--is equal to the difference of the proportional growth rates of the dividend (E) and the divisor (Q).

3. The proportional growth rate of a quantity raised to a exponent--Q^y, say--is equal to the exponent (y) times the growth rate of the quantity (Q).

Using these rules of thumb, it is easy to see that if the economy's total capital stock K are increasing at the proportional rate of growth n +

g—the sum of the rates of growth of labor efficiency and the labor force—then the economy's total output Y will be increasing at that same proportional rate, that the capital-output ratio will be constant, and the economy will be in equilibrium on its *balanced-growth path.*

Recall the Cobb Douglas production function:

$$Y_t = \left(K_t\right)^{\alpha} \times \left(L_t\right)^{1-\alpha} \times \left(E_t\right)^{1-\alpha}$$

Output is the product of three terms, therefore its proportional growth rate is the sum of the growth rates of those three terms. Each of the individual terms is a quantity raised to a power, therefore each individual term's growth rate is the growth rate of the quantity (the capital stock K, the labor force L, or labor efficiency E) times the appropriate power (α, or 1-α).

Writing g(y) to express the proportional growth rate of output Y, we can write this in algebra as:

$$g(y) = \alpha \times g(k) + (1-\alpha) \times g(l) + (1-\alpha) \times g(e)$$

We know that the growth rate of labor efficiency g(e) = g, and that the growth rate of the labor force g(l) = n. So if it is the case that the growth rate of the capital stock g (k) = n+g, then we can substitute into the equation above and conclude that:

$$g(y) = \alpha \times (n+g) + (1-\alpha) \times n + (1-\alpha) \times g$$
$$g(y) = n + g$$

So if the capital stock is growing at a rate n+g, then total output is growing at the same rate n+g. Since output and capital are growing at the same rate, this means that the capital-output ratio κ is constant.

Thus the economy is in equilibrium along its balanced-growth path.

The Equilibrium Capital-Output Ratio κ*

What must the capital-output ratio be for the capital stock to be growing at the proportional rate n+g? Remember our expression for what next year's capital stock would be:

$$K_{t+1} = K_t + (s \times Y_t) - (\delta \times K_t)$$

We can turn this into an expression for the proportional growth rate of the capital stock by subtracting this year's capital stock K_t from both sides, and then dividing by this year's capital stock K_t:

$$g(k) = \frac{K_{t+1} - K_t}{K_t} = s \times \frac{Y_t}{K_t} - \delta$$

Box 4.4: An Example: The Growth Rate of the Capital Stock

Suppose that the depreciation rate δ were 4 percent per year and the savings rate were 20 percent. We can then calculate what the proportional rate of growth of the capital stock would be for each possible level of the capital-output ratio.

Begin with the equation for the growth rate of the capital stock:

$$g(k_t) = s \times \frac{Y_t}{K_t} - \delta$$

But multiplying by the output-to-capital ratio Y/K is the same thing as dividing by the capital-output ratio κ:

$$g(k_t) = \frac{s}{\kappa_t} - \delta$$

Simply substitute the values of the depreciation rate and the savings share into this equation to get:

$$g(k_t) = \frac{0.20}{\kappa_t} - 0.04$$

Then if the current capital-output ratio were to be five, the growth rate of the capital stock would be:

$$g(k_t) = \frac{0.20}{5} - 0.04 = 0$$

zero: the capital stock would be constant.

By contrast, if the current capital-output ratio were 2.5, the growth rate of the capital stock would be:

$$g(k_t) = \frac{0.20}{2.5} - 0.04 = 0.04$$

plus 2 percent per year: the capital stock would be growing at 4 percent per year.

Then if we set the proportional growth rate of the capital stock to n+g:

$$n + g = s \times \frac{Y_t}{K_t} - \delta$$

add the depreciation rate to both sides:

$$n + g + \delta = s \times \frac{Y_t}{K_t}$$

divide both sides by the savings rate:

$$\frac{n + g + \delta}{s} = \frac{Y_t}{K_t}$$

flip the equation:

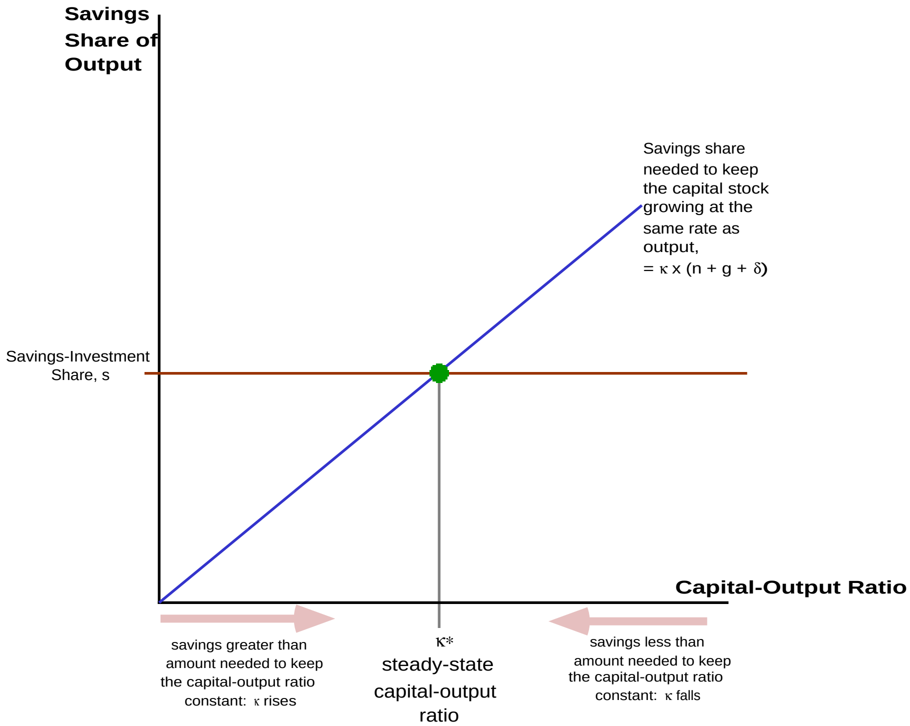
$$\frac{K_t}{Y_t} = \frac{s}{n + g + \delta}$$

and remember that K/Y is the capital-output ratio κ:

$$\frac{K_t}{Y_t} = \kappa = \frac{s}{n + g + \delta} = \kappa^*$$

We discover that the capital stock is growing at the rate n+g—and the economy is on its balanced-growth equilibrium—when the capital-output ratio is at its steady-state value κ*, equal to s/(n+g+δ).

Figure 4.10: Why κ* Is the Equilibrium Capital-Output Ratio



If the current capital-output ratio is equal to its steady-state value κ*, then the share of output saved and invested every year is exactly what is needed to keep the capital stock growing at the same rate as output, and keep the capital-output ratio constant.

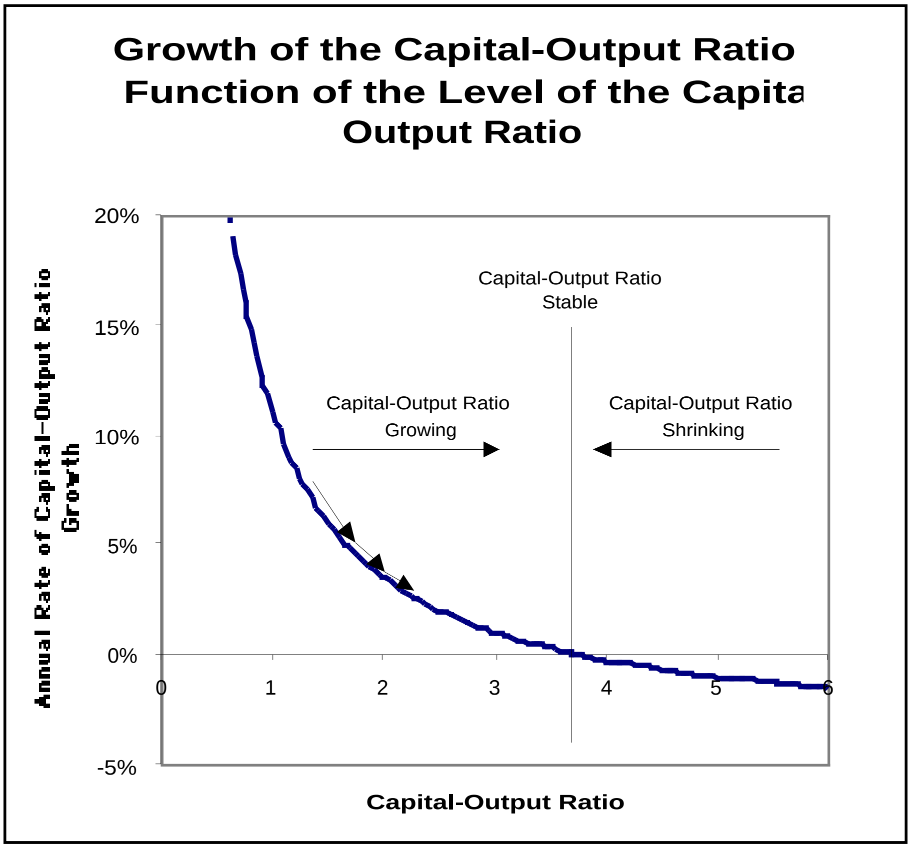
Balanced-Growth Equilibrium

If the capital-output ratio κ is below κ*, the share of output invested each year (equal to s) is greater than needed to keep the capital stock growing as fast as output (equal to κ(n + g + δ)). The capital-output ratio rises. If it is above κ*, the share invested each year (equal to s) is less than needed to keep the capital stock growing as fast as output (equal to κ(n + g + δ)). The capital-output ratio falls. The economy closes some of the gap between its current position and its steady-state growth path. Only if it is equal to its steady state value:

$$\kappa^* = \frac{s}{n+g+\delta}$$

will the capital-output ratio be constant.

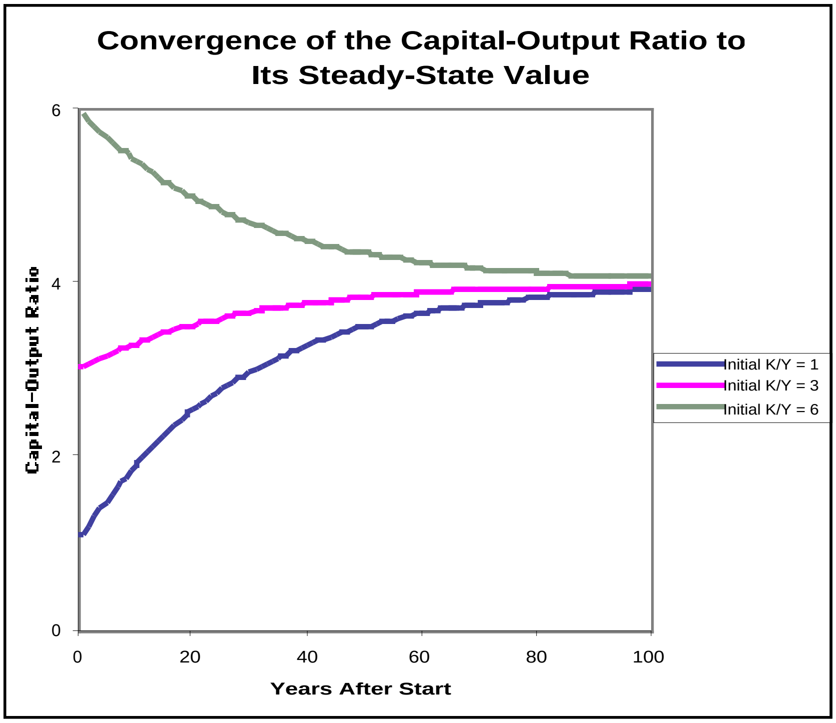
Figure 4.11: Dynamics of the Capital-Output Ratio



The value of the capital-output ratio at which its rate of change is zero is an *equilibrium.* If the capital-output ratio is at that *equilibrium* value, it will stay there. If it is away from that equilibrium value, it will head toward it.

Thus the value s/(n+g+δ) is the *equilibrium level* of the capital-output ratio. It is a point at which the economy tends to balance, and to which the economy converges. The requirement that the capital-output ratio equal this equilibrium level becomes our equilibrium condition for balanced economic growth.

Figure 4.12: Convergence of the Capital-Output Ratio



If the capital-output ratio starts at a value different from its steady-state equilibrium value, it will head towards equilibrium. The figure shows the paths over time of the capital-output ratio for parameter values of s=0.28, n=0.02, g=0.015, δ=0.035, α=0.5, and for different initial starting values of 1, 3, and 6. The steady-state capital-output ratio κ* is 4.

Output per Worker On the Steady State Growth Path

When the capital-output ratio is at its steady-state balanced-growth equilibrium value κ*, we say that the economy is on its steady-state growth path. What is the level of output per worker if the economy is on its steady-state growth path? We saw the answer to this this back in Chapter 3. The requirement that the economy be on its steady-state growth path was then our equilibrium condition:

$$\left(\frac{K_t}{Y_t}\right) = \kappa^* = \frac{s}{n + g + \delta}$$

And in order to combine it with the production function:

$$(Y_t / L_t) = (K_t / L_t)^{\alpha} \times (E_t)^{1-\alpha}$$

we first rewrote the production function to make capital-per-worker the product of the capital-output ratio and output per worker:

$$(Y_t / L_t) = (Y_t / L_t \times K_t / Y_t)^{\alpha} \times (E_t)^{1-\alpha}$$

Dividing both sides by $(Y/L)^{\alpha}$:

$$(Y_t / L_t)^{1-\alpha} = (K_t / Y_t)^{\alpha} \times (E_t)^{1-\alpha}$$

Raising both sides to the 1/(1-α) power produces an equation for the level of output per worker:

$$(Y_t / L_t) = (K_t / Y_t)^{\left(\frac{\alpha}{1-\alpha}\right)} \times (E_t)$$

Substitute the equilibrium condition into this transformed form of the production function. The result is that, as long as the economy is on balanced-growth path:

$$(Y_t / L_t) = \kappa *^{\left(\frac{\alpha}{1-\alpha}\right)} \times E_t = \left(\frac{s}{n+g+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} \times E_t$$

If we define:

$$\lambda = \frac{\alpha}{1-\alpha}$$

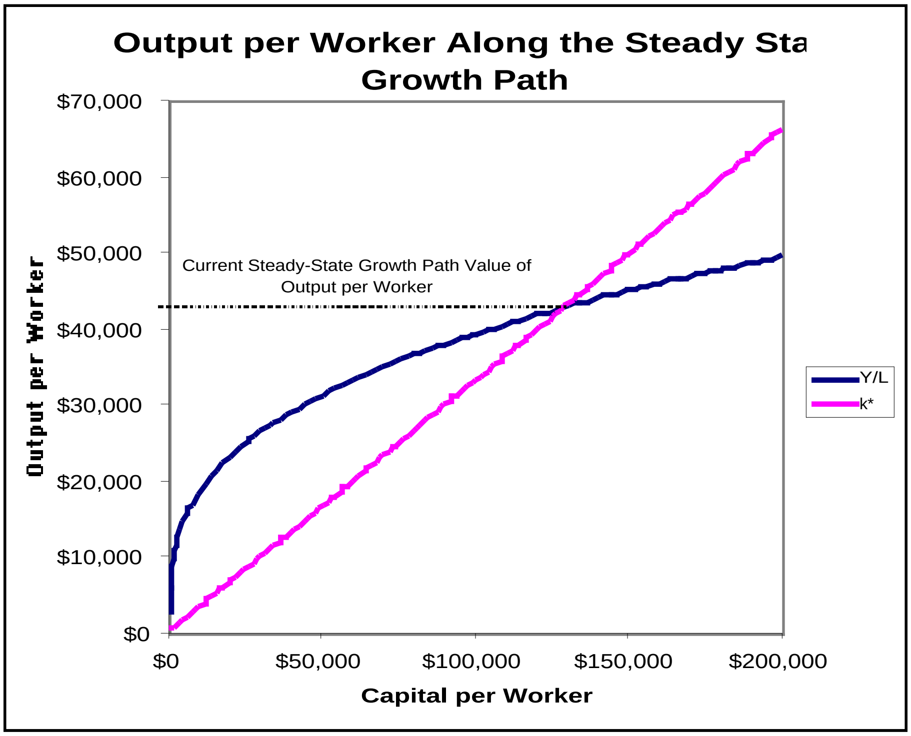
and call λ the *growth multiplier* (where does the growth multiplier arise from? See Box 4.5), then output per worker along the steady-state growth path is equal to the steady-state capital-output ratio raised to the growth multiplier, times the current level of the efficiency of labor:

$$\left(\frac{Y_t}{L_t}\right)_{ss} = \kappa *^{\lambda} \times E_t$$

Notes—and this is very important—that the balanced-growth path level of output per worker is not constant. As time passes, the balanced-growth path level of output per worker rises. Why? Because output per worker Y/L is equal to the *current* efficiency of labor E_t times the steady-state capital-output ratio κ* raised to the power λ. Technological progress means that the efficiency of labor E_t grows at a proportional growth rate g.

We call this equilibrium a *balanced-growth equilibrium* because all the macroeconomic quantities (save the capital-output ratio itself) are growing in a balanced fashion. The labor force is growing at the rate equal to n. Both the efficiency of labor and output per worker are growing at the rate g. And both the capital stock and total output are growing at the rate n+g.

Figure 4.13: Calculating Steady-State Output per Worker



Is the algebra too complicated? There is an alternative, diagrammatic way of seeing what the steady-state capital-output ratio implies for the steady-state level of output per worker. Simply draw the production function for the current level of the efficiency of labor E_t. Also draw the line that shows where the capital-output ratio is equal to its steady state value, κ^*. Look at the point where the curves intersect. That point shows what the current level of output per worker is along the steady state growth path (for the current level of the efficiency of labor).

Anything that increases the steady-state capital-output ratio will rotate the capital-output line to the right. Thus it will raise steady-state output per worker. Anything that decreases the steady-state capital-output ratio rotates the capital-output line to the left. It thus lowers steady-state output per worker.

Thus calculating output per worker when the economy is on its steady-state growth path is a simple three-step procedure:

- First, calculate the steady-state capital-output ratio, $\kappa^*=s/(n+g+\delta)$, the savings rate divided by the sum of the population growth rate, the efficiency of labor growth rate, and the depreciation rate.
- Second, amplify the steady-state capital-output ratio κ^* by the growth multiplier. Raise it to the $\lambda = (\alpha/(1-\alpha))$ power, where α is the diminishing-returns-to-capital parameter.
- Third, multiply the result by the current value of the efficiency of labor E_t, which can be easily calculated because the efficiency of labor is growing at the constant proportional rate g.

And the fact that an economy converges to its steady-state growth path makes analyzing the long-run growth of an economy relatively easily as well:

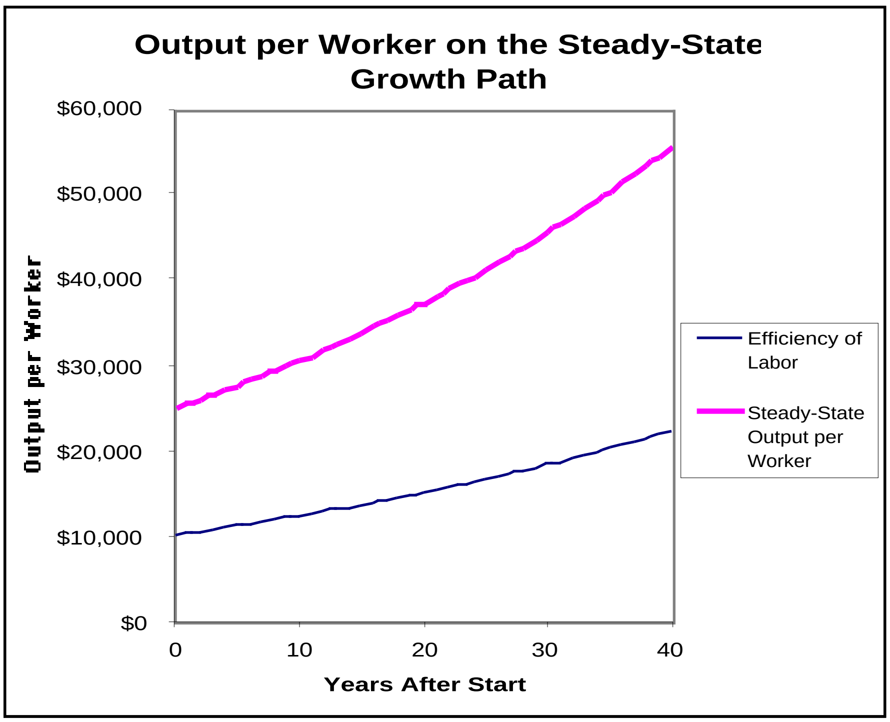
- First calculate the steady-state growth path.
- From the steady-state growth path, forecast the future of the economy: If the economy is on its steady-state growth path today,

it will stay on its steady-state growth path in the future (unless some of the parameters--n, g, δ, s, and α—shift).

- If the economy is not on its steady-state growth path today, it is heading for its steady-state growth path and will get there soon.

Thus long-run economic forecasting becomes simple.

Figure 4.14: Output per Worker On the Steady-State Growth Path



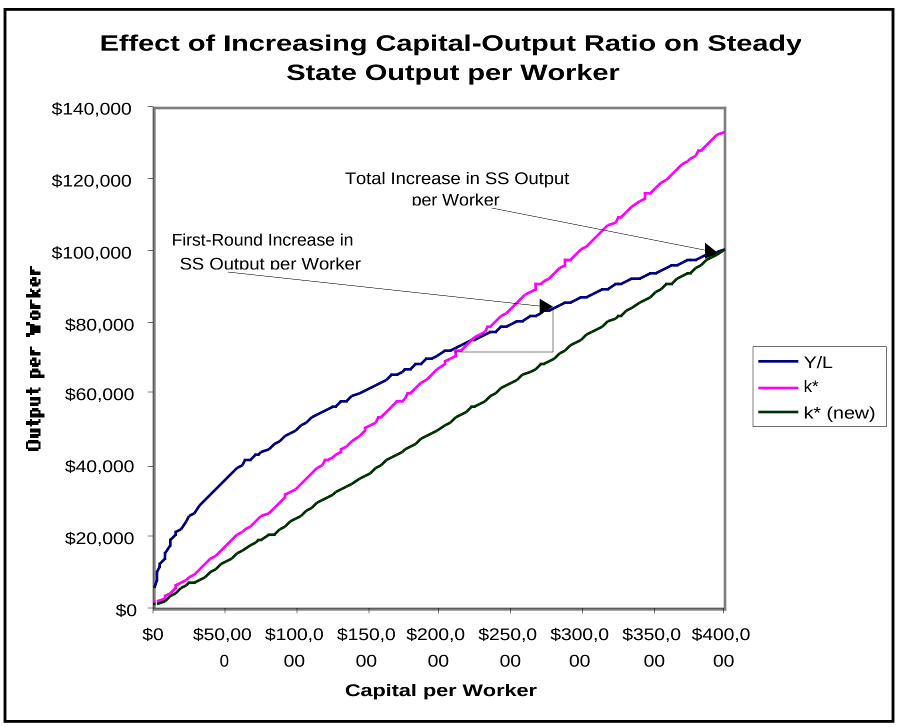
Parameter values: labor-force growth rate n at 1% per year; increase in the efficiency of labor g at 2% per year; depreciation rate δ at 3% per year; savings rate s at 37.5%; and diminishing-returns-to-capital parameter α at 1/3. The efficiency of labor and output per worker grow smoothly along the economy's balanced growth path.

Box 4.5: Details: Where the Growth Multiplier Comes From

Why is the steady-state capital-output ratio raised to the (larger) power of $(\alpha/(1-\alpha))$ rather than just the power α? It makes a big difference when one applies the growth model to different situations.

The reason is that an increase in the capital-output ratio increases the capital stock both directly and indirectly. For the same level of output you have more capital. And because extra output generated by the additional capital is itself a source of additional savings and investment, you have higher capital for that reason as well. The "impact" effect of the additional capital generated by anything that raises __--an increase in savings, or a decrease in labor force, or anything else--is thus multiplied by these positive feedback effects.

Figure 4.15: The Growth Multiplier



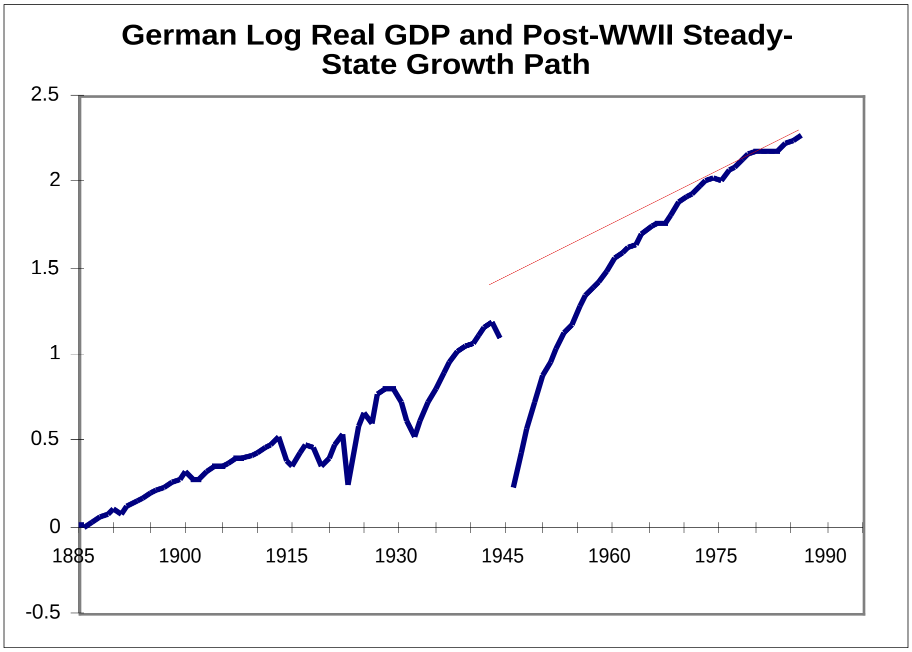
The figure above shows the effect of this difference between α and $(\alpha/(1-\alpha))$. An increase in the capital-output ratio means more capital for a given level of output, and that generates the first-round increase in output: amplification by the increase in capital raised to the power α. But the first-round increase in output generates still more capital, which increases production further. The total increase in production is the proportional increase in the steady-state capital-output ratio raised to the (larger) power $(\alpha/(1-\alpha))$.

How Fast Does the Economy Head For Its Steady-State Growth Path?

Suppose that the capital-output ratio κ_t is not at its steady state value κ^*? How fast does it approach its steady state? It would take too long and be too complicated for us to derive it here, but nevertheless it is a fact that, under the Cobb-Douglas production function, the growth rate of the capital-output ratio will be equal to a fraction $(1-\alpha) \times (n+g+\delta)$ of the gap between the steady-state and its current level.

For example, if $(1-\alpha) \times (n+g+\delta)$ is equal to 0.04, the capital-output ratio will close approximately 4 percent of the gap between its current level and its steady-state value in a year. If $(1-\alpha) \times (n+g+\delta)$ is equal to 0.07, the capital-output ratio closes 7 percent of the gap between its current level and its steady-state value in a year. A variable closing 4 percent of the gap each year between its current and its steady-state value will move halfway to its steady-state value in 18 years, and three-quarters of the way to its steady-state value in 36 years. A variable closing 7 percent of the gap each year between its current and its steady-state value will move halfway to its steady-state value in 10 years, and will move three-quarters of the way to its steady-state value in 20 years.

Figure 4.16: West German Convergence to Its Steady-State Growth Path



The end of World War II left the West German economy in ruins. Yet within twelve years it had closed half the gap back to its steady-state growth path. And within thirty years it had closed effectively all of the gap back to its steady-state growth path. Economists study equilibrium steady-state growth paths for a reason: economies do converge to them and then remain on them.

This fact allows us to make much better medium-run forecasts of the dynamic of the economy:

- An economy that is not on its steady-state growth path will close a fraction $(1-\alpha) \times (n+g+\delta)$ of the gap between its current state and its steady-state growth path in a year.

Box 4.6: Example: Converging to the Steady-State Balanced-Growth Path

Thus an economy with parameter values of population growth n=0.02, efficiency of labor growth g=0.015, depreciation δ=0.035, and a diminishing-returns-to-investment parameter α=0.5 would, if off of its steady-state growth path, close a fraction:

$$(1-\alpha)\times(n+g+\delta) = (1-0.5)\times(.02+.015+.035)$$
$$= 0.5\times.07 = 0.035$$

of 3.5 percent of the gap between its current state and its steady-state each year. Such a rate of convergence would allow the economy to close half of the gap to the steady-state in twenty years.

Thus short- and medium-run forecasting becomes simple too. All you have to do is to predict that the economy will head for its steady-state growth path, and calculate what the steady-state growth path is.

Determining the Steady-State Capital-Output Ratio

Labor Force Growth

The faster the growth rate of the labor force, the lower will be the economy's steady-state capital-output ratio. Why? Because each new worker who joins the labor force must be equipped with enough capital to be productive, and to on average match the productivity of his or her peers.

The faster the rate of growth of the labor force, the larger the share of current investment that must go to equip new members of the labor force with the capital they need to be productive. Thus the lower will be the amount of investment that can be devoted to building up the average ratio of capital to output.

Box 4.7: Example: An Increase in Population Growth

Consider an economy in which the parameter α is 1/2--so that the growth multiplier $\lambda = (\alpha/(1-\alpha))$ is one--in which the underlying rate of productivity growth g is 1.5% per year, the depreciation rate δ is 3.5% per year, and the savings rate s is 21%. Suppose that the labor force growth rate suddenly and permanently increases from one to two percent per year.

Then before the increase in population growth the steady-state capital output ratio was:

$$\kappa *_{old} = \frac{s}{n_{old} + g + \delta} = \frac{.21}{.01 + .015 + .035} = \frac{.21}{.06} = 3.5$$

After the increase in population growth, the new steady-state capital-output ratio will be:

$$\kappa *_{new} = \frac{s}{n_{new} + g + \delta} = \frac{.21}{.02 + .015 + .035} = \frac{.21}{.07} = 3$$

Before the increase in population growth, the level of output per worker along the old steady-state growth path was:

$$\left(Y_t / L_t\right)_{ss,old} = (\kappa *)^{\lambda} \times E_t = (3.5)^1 \times E_t$$

After the increase in population growth, the level of output per worker along the new steady-state growth path will be:

$$\left(Y_t / L_t\right)_{ss,new} = (\kappa *)^{\lambda} \times E_t = (3.0)^1 \times E_t$$

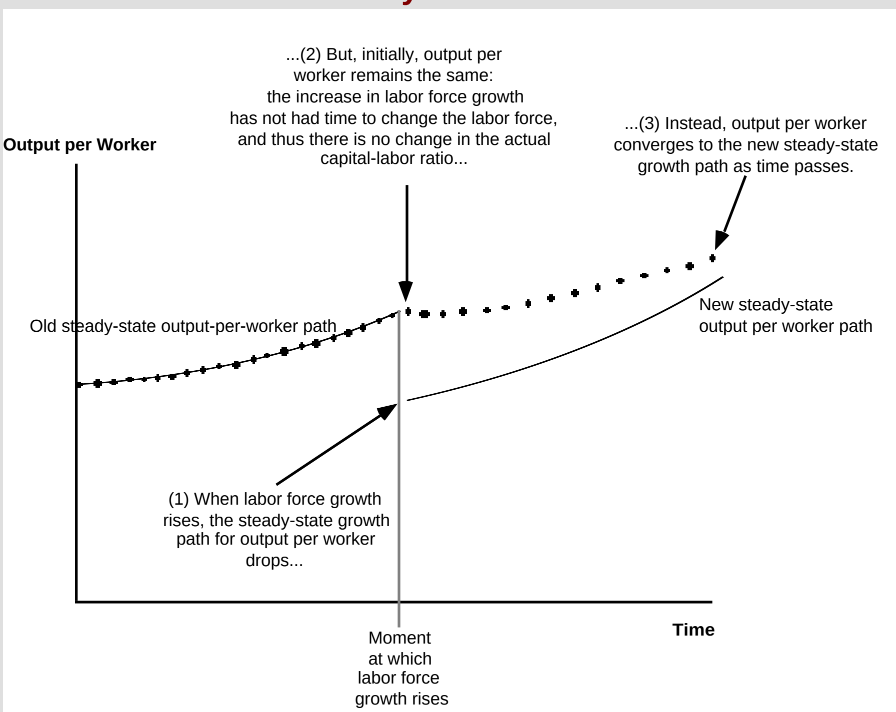
Divide the second of the equations by the first

$$\frac{\left(Y_t / L_t\right)_{ss,new}}{\left(Y_t / L_t\right)_{ss,old}} = \frac{(3.0)^1 \times E_t}{(3.5)^1 \times E_t} = 0.857$$

And discover that output per worker along the new steady-state growth path is only 86% of what it would have been along the old steady-state growth path: faster population growth means that output per worker along the steady-state growth path has fallen by 14 percent.

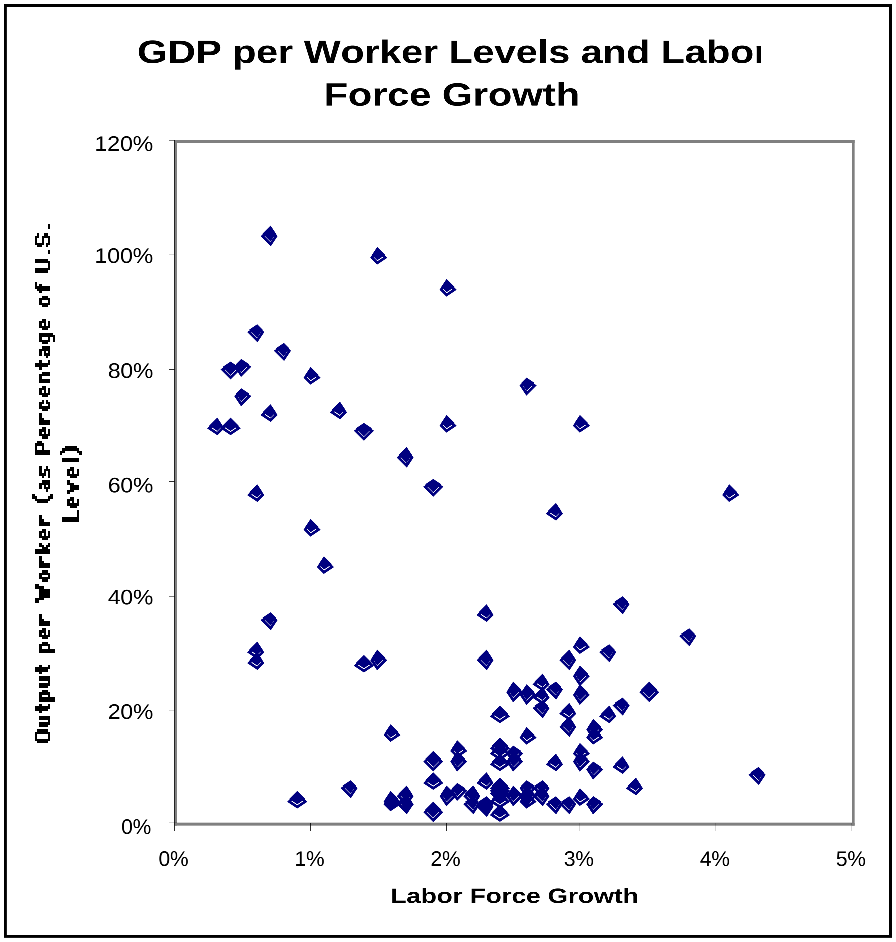
In the short run this increase in labor force growth will have no effect on output per worker. Just after population growth increases, the increased rate of population growth has had no time to increase the population. It has had no time to affect the actual capital-labor ratio. But over time the economy will converge to the new, lower, steady-state growth path, and output per worker will be reduced by 14% relative to what it would otherwise have been.

Figure 4.18: Effects of a Rise in Population Growth on the Economy's Growth Path



Although a sudden change in one of the parameters of the economic growth model causes a sudden change in the location of the economy's steady-state growth path, the economy's level of output per worker does not instantly jump to the new steady-state value. Instead, it converges to the new steady-state value only slowly, over considerable periods of time.

Figure 4.17: Labor Force Growth and GDP per Worker Levels



The average country with a labor force growth rate of less than one percent per year has an output per worker level nearly 60% of the U.S. level. The average country with a labor force growth rate of more than three percent per year has an output per worker level only 20% of the U.S. level. Not all of this is due toa one way relationship from fast population growth to high investment requirements to low steady-state capital-output ratios: countries are not just poor because they have fast labor force growth rates, to some degree they have fast labor force growth rates because they are poor. But some of it is. High labor force growth rates are a powerful cause of relative poverty in the world today.

Source: Author's calculations from the Penn World Table data constructed by Alan Heston and Robert Summers, online at http://www.nber.org.

A sudden and permanent increase in the rate of growth of the labor force will lower the level of output per worker on the steady-state growth path. How large will the long-run change in the level of output be, relative to what would have happened had population growth not increased? It is straightforward to calculate if we know what the other parameter values of the economy are.

How important is all this in the real world? Does a high rate of labor force growth play a role in making countries relatively poor not just in economists' models but in reality? It turns out that it is important, as Figure 4.17 shows. Of the twenty-two countries in the world with GDP per worker levels at least half that of the U.S. level, eighteen have labor force growth rates of less than 2% per year, and twelve have labor force growth rates of less than 1% per year. The additional investment requirements imposed by rapid labor force growth are a powerful reducer of capital intensity, and a powerful obstacle to rapid economic growth.

Depreciation and Productivity Growth

Increases or decreases in the depreciation rate will have the same effects on the steady-state capital-output ratio and on output per worker along the steady-state growth path as increases or decreases in the labor force growth rate. The higher the depreciation rate, the lower will be the economy's steady-state capital-output ratio. Why? Because a higher depreciation rate means that the existing capital stock wears out and must be replaced more quickly. The higher the depreciation rate, the larger the share of current investment that must go replace the capital that has become worn-out or obsolete. Thus the lower will be the amount of investment that can be devoted to building up the average ratio of capital to output.

Increases or decreases in the rate of productivity growth will have similar effects as increases or decreases in the labor force growth rate on the steady-state capital-output ratio, but they will have very different effects on the steady-state level of output per worker. The faster the growth rate of productivity, the lower will be the economy's steady-state capital-output ratio. The faster is productivity growth, the higher is output now. But the capital stock depends on what investment was in the past. The faster is productivity growth, the smaller is past investment relative to current production, and the lower is the average ratio of capital to output. So a change in productivity growth will have the same effects on the steady-state capital-output ratio as an equal change in labor force growth.

But a change in productivity growth will have very different effects on output per worker along the steady-state growth path. Output per worker along the steady-state growth path is:

$$\left(Y_t / L_t\right)_{ss} = (\kappa\ *)^{\lambda} \times E_t$$

While an increase in the productivity growth rate g lowers κ*, it increases the rate of growth of the efficiency of labor E, and so in the long run it does not lower but raises output per worker along the steady-state growth path.

The Savings Rate

The higher the share of national product devoted to savings and gross investment, the higher will be the economy's steady-state capital-output ratio. Why? Because more investment increases the amount of new capital that can be devoted to building up the average ratio of capital to output. Double the share of national product spent on gross investment, and you will find that you have doubled the economy's capital intensity--doubled its average ratio of capital to output.

One good way to think about it is that the steady-state capital-output ratio is that at which the economy's investment effort and its investment requirements are in balance. Investment effort is simply s, the share of total output devoted to savings and investment. Investment requirements are the amount of new capital needed to replace depreciated and worn out machines and buildings (a share of total output equal to δ x κ^*), plus the needed to equip new workers who increase the labor force (a share of total output equal to n x κ^*), plus the amount needed to keep the stock of tools and machines at the disposable of workers increasing at the same rate as the efficiency of their labor (a share of total output equal to g x κ^*). So double the savings rate and you double the steady-state capital-output ratio.

Box 4.8: Example: An Increase in the Savings Rate

For an example of how an increase in savings changes output per worker along the steady-state growth path, consider an economy in which the parameter α is 1/2--so that $\lambda = (\alpha/(1-\alpha))$ is one--in which the underlying rate of labor force growth is 1% per year, the rate of productivity growth g is 1.5% per year, the depreciation rate δ is 3.5% per year. Suppose that the savings rate s was 18%, and suddenly and permanently rises to 24%.

Then before the increase in savings, the steady-state capital output ratio was:

$$\kappa *_{old} = \frac{s_{old}}{n+g+\delta} = \frac{.18}{.01+.015+.035} = \frac{.18}{.06} = 3.0$$

After the increase in savings, the new steady-state capital-output ratio will be:

$$\kappa *_{new} = \frac{s_{new}}{n+g+\delta} = \frac{.24}{.01+.015+.035} = \frac{.24}{.06} = 4$$

Before the increase in savings, the level of output per worker along the old steady-state growth path was:

$$\left(Y_t / L_t\right)_{ss,old} = (\kappa *)^{\lambda} \times E_t = (3.0)^1 \times E_t$$

After the increase in savings, the level of output per worker along the new steady-state growth path will be:

$$\left(Y_t / L_t\right)_{ss,new} = (\kappa *)^{\lambda} \times E_t = (4.0)^1 \times E_t$$

Divide the second of the equations by the first

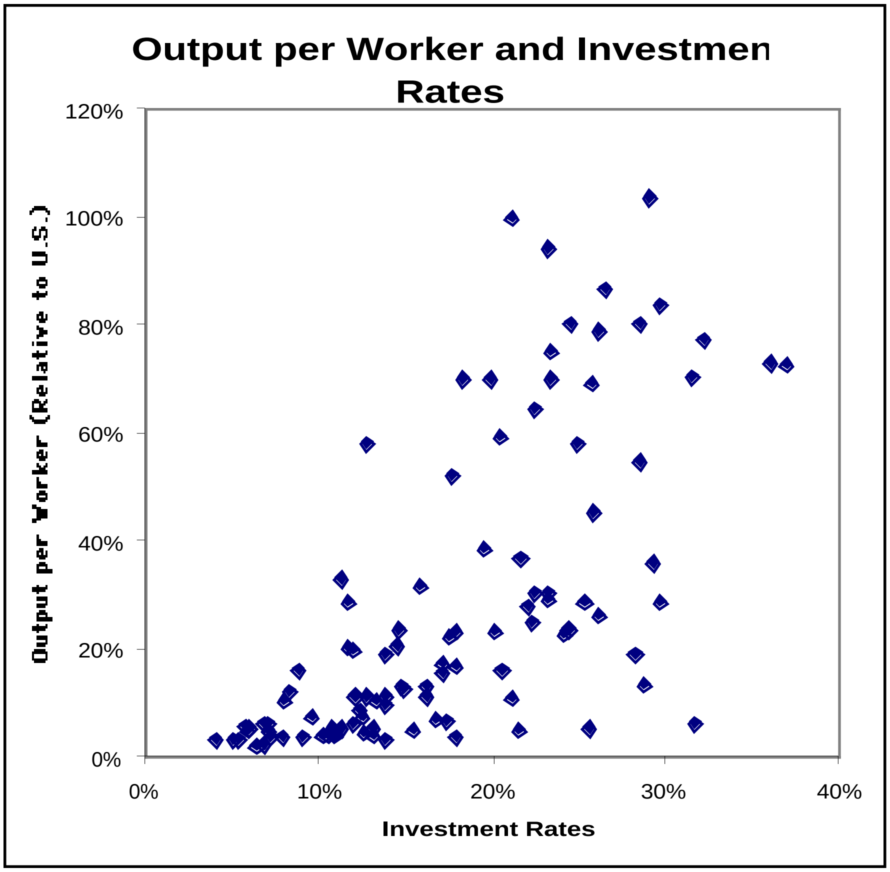
$$\frac{\left(Y_t / L_t\right)_{ss,new}}{\left(Y_t / L_t\right)_{ss,old}} = \frac{(4.0)^1 \times E_t}{(3.0)^1 \times E_t} = 1.333$$

And discover that output per worker along the new steady-state growth path is 133% of what it would have been along the old steady-state growth path: higher savings means that output per worker along the steady-state growth path has risen by 33 percent.

The increase in savings has no effect on output per worker immediately. Just after the increase in savings has taken place the economy is still on its old, lower steady-state growth path. But as time passes it converges to the new steady-state growth path corresponding to the higher level of savings, and in the end output per worker is 33 percent higher than it would otherwise have been.

How important is all this in the real world? Does a high rate of savings and investment play a role in making countries relatively rich not just in economists' models but in reality? It turns out that it is important indeed, as Figure 4.16 shows. Of the twenty-two countries in the world with GDP per worker levels at least half that of the U.S. level, nineteen have investment shares of more than 20% of output. The high capital-output ratios generated by high investment efforts are a very powerful source of relative prosperity in the world today.

Figure 4.19: National Investment Shares and GDP per Worker Levels



The average country with an investment share of output of more than twenty-five percent has an output per worker level more than 70% of the U.S. level. The average country with an investment share of output of less than fifteen percent has an output per worker level less than 15% of the U.S. level. Not all of this is due to a one way relationship from a high investment effort to a high steady-state capital-output ratio: countries are not just poor because they invest little, to some degree they invest little because they are poor. But much of it is. High savings and investment rates are a very powerful cause of relative wealth in the world today.

Source: Author’s calculations from the Penn World Table data constructed by Alan Heston and Robert Summers, online at http://www.nber.org.

Recap 4.4: Understanding the Growth Model

A few rules of thumb help us understand the growth model. Double the savings rate and you double the steady-state capital-output ratio, and increase the level of GDP per worker by a factor of 2 raised to the (α/(1- α)) power. An increase in the population growth rate lowers the steady-state capital-output ratio by an amount proportional to its increase in the economy's investment requirements--the sum of depreciation, labor force growth, and efficiency of labor growth. As it lowers the steady-state capital-output ratio it lowers the steady-state growth path of output per worker as well. An increase in the efficiency of labor growth rate lowers the steady-state capital-output ratio, but raises the steady-state growth path of output per worker.

4.5 Chapter Summary

Main Points

1. One principal force driving long-run growth in output per worker is the set of improvements in the efficiency of labor springing from technological progress

2. A second principal force driving long-run growth in output per worker are the increases in the capital stock which the average worker has at his or her disposal and which further multiplies productivity.

3. An economy undergoing long-run growth converges toward and settles onto an equilibrium steady-state growth path, in which the economy's capital-output ratio is constant.

4. The steady -state level of the capital-output ratio is equal to the economy's savings rate, divided by the sum of its labor force growth rate, labor efficiency growth rate, and depreciation rate.

Important Concepts

Production Function
Capital
Output per Worker
Efficiency of Labor
Capital-Output Ratio
Steady-State Growth Path
Savings Rate
Depreciation Rate
Labor Force
Convergence
Consumption per Worker
"Golden Rule" Savings Rate

Analytical Exercises

1. Consider an economy in which the depreciation rate is 3% per year, the rate of population increase is 1% per year, the rate of technological progress is 1% per year, and the private savings rate is 16% of GDP. Suppose that the government increases its budget deficit--which had been at 1% of GDP for a long time--to 3.5% of GDP and keeps it there indefinitely.

What will be the effect of this shift in policy on the economy's steady-state capital-output ratio?

What will be the effect of this shift in policy on the economy's steady state growth path for output per worker? How does your answer depend on the value of the diminishing-returns-to-capital parameter α?

Suppose that your forecast of output per worker 20 years in the future had been \$100,000. What is your new forecast of output per worker twenty years hence?

2. Suppose that a country has the production function:

$$Y_t = (K_t)^{0.5} \times (E_t \times L_t)^{0.5}$$

What is output Y considered as a function of the level of the efficiency of labor E, the size of the labor force L, and the capital-output ratio (K/Y)?

What is output per worker Y/L?

3. Suppose that with the production function:

$$Y_t = (K_t)^{0.5} \times (E_t \times L_t)^{0.5}$$

the depreciation rate on capital is three percent per year, the rate of population growth is one percent per year, and the rate of growth of the efficiency of labor is one percent per year.

Suppose that the savings rate is ten percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path written as a function of the level of the efficiency of labor?

Suppose that the savings rate is fifteen percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path?

Suppose that the savings rate is twenty percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path?

4. What happens to the steady-state capital-output ratio if the rate of technological progress increases? Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain in the same position?

5. Discuss the following proposition: "An increase in the savings rate will increase the steady-state capital output ratio, and so increase both output per worker and the rate of economic growth in both the short run and the long run."

6. Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain the same if capital were to become more durable--if the rate of depreciation on capital were to fall?

7. Suppose that a sudden disaster--an epidemic, say--reduces a country's population and labor force, but does not affect its capital stock. Suppose further that the economy was on its steady-state growth path before the epidemic.

What is the immediate effect of the epidemic on output per worker?

On the total economy-wide level of output?

What happens subsequently?

8. According to the marginal productivity theory of distribution, in a competitive economy the rate of return on a dollar's worth of capital--

its profits or interest--is equal to capital's marginal productivity. With the production function:

$$\left(\frac{Y_t}{L_t}\right) = \left(\frac{K_t}{L_t}\right)^{\alpha} (E_t)^{1-\alpha}$$

what is the marginal product of capital? How much is total output (Y, not Y/L) boosted by the addition of an extra unit to the capital stock?

9. According to the marginal productivity theory of distribution, in a competitive economy the rate of return on a dollar's worth of capital--its profits or interest--is equal to capital's marginal productivity. If this theory holds and the marginal productivity of capital is indeed:

$$dY/dK = \alpha \times (Y/K)$$

How large are the total earnings received by capital? What share of total output will be received by the owners of capital as their income?

10. Suppose that environmental regulations lead to a slowdown in the rate of growth of the efficiency of labor in the production function, but also lead to better environmental quality. Should we think of this as a "slowdown" in economic growth or not?

Policy-Relevant Exercises

1. In the mid-1990s during the Clinton Presidency the U.S. eliminated its federal budget deficit. The national savings rate was thus boosted by 4% of GDP, from 16% to 20% of real GDP. In the U.S. in the mid-1990s, the rate of labor force growth was 1% per year, the depreciation rate was 3% per year, the rate of increase of the efficiency of labor was 1% per year, and that the diminishing-returns-to-capital parameter α is 1/3. Suppose that these rates continue into the indefinite future.

    Suppose that the federal budget deficit had remained at 4% indefinitely. What then would have been the U.S. economy's steady-state capital-output ratio? If the efficiency of labor in 2000 were $30,000 per year, what would have been your forecast of output per worker in the U.S. in 2040?

After the elimination of the federal budget deficit, what would be your calculation of been the U.S. economy's steady-state capital-output ratio? If the efficiency of labor in 2000 were $30,000 per year, what would have been your forecast of output per worker in the U.S. in 2040?

2. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 1/2, and if your estimate of the efficiency of labor in 2000 were not $30,000 but $15,000 a year?

3. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 2/3?

4. What are the long-run costs as far as economic growth is concerned of a policy of taking money that would reduce the national debt—and thus add to national savings—and distributing it as tax cuts instead? What would be the long-run benefits of such a policy? How could we decide whether such a policy was a good thing or not?

5. At the end of the 1990s it appeared that because of the computer revolution the rate of growth of the efficiency of labor in the United States had doubled, from 1 percent per year to 2 percent per year. Suppose this increase were to be permanent. And suppose the rate of labor force growth were to remain constant at 1 percent per year, the depreciation rate were to remain constant at 3 percent per year, and the American savings rate (plus foreign capital invested in America) were to remain constant at 20 percent per year. Assume that the efficiency of labor in the U.S. in 2000 is $15,000 per year, and that the diminishing-returns-to-capital parameter α is 1/3.

What is the change in the steady-state capital-output ratio?

What is the new capital-output ratio?

What would such a permanent acceleration in the rate of growth of the efficiency of labor change your forecast of the level of output per worker in 2040?

6. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 1/2, and if your estimate of the efficiency of labor in 2000 were not $30,000 but $15,000 a year?

7. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 2/3?

8. Output per worker in Mexico in the year 2000 is about $10,000 per year. Labor force growth is 2.5% per year. The depreciation rate is 3% per year. The rate of growth of the efficiency of labor is 2.5% per year. The savings rate is 16% of GDP. And the diminishing-returns-to-capital parameter α is 0.5.

What is Mexico's steady-state capital-output ratio?
Suppose that Mexico today is on its steady-state growth path.
What is the current level of the efficiency of labor E?
What is your forecast of output per worker in Mexico in 2040?

9. In the framework of the question above…

…how much does your forecast of output per worker in Mexico in 2040 increase if Mexico's domestic savings rate remains unchanged but it is able to finance extra investment equal to 4% of GDP every year by borrowing from abroad?
…how much does your forecast of output per worker in Mexico in 2040 increase if the labor force growth rate immediately falls to 1% per year?
…how much does your forecast of output per worker in Mexico in 2040 increase if both happen?

10. Consider an economy with a labor force growth rate of 2% per year, a depreciation rate of 4% per year, a rate of growth of the efficiency of labor of 2% per year, and a savings rate of 16% of GDP.

Suppose that the diminishing-returns-to-capital parameter α is 1/3. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?
Suppose that the diminishing-returns-to-capital parameter α is 1/2. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?
Suppose that the diminishing-returns-to-capital parameter α is 2/3. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?
Suppose that the diminishing-returns-to-investment capital α is 3/4. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?