PART II: LONG-RUN ECONOMIC GROWTH
Chapter 4: The Theory of Economic Growth

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Questions

1. What are the principal determinants of long-run economic growth?

2. What equilibrium condition is useful in analyzing long-run growth?

3. How quickly does an economy head for its steady-state growth path?

4. What effect does faster population growth have on long-run growth?

5. What effect does a higher savings rate have on long-run growth?

4.1 Sources of Long-Run Growth

Ultimately, long-run economic growth is *the* most important aspect of how the economy performs. Material standards of living and levels of economic productivity in the United States today are about four times what they are today in, say, Mexico because of favorable initial conditions and successful growth-promoting economic policies over the past two centuries. Material standards of living and levels of economic productivity in the United States today are at least five times what they were back at the end of the nineteenth century, and more than ten times what they were back at the founding of the republic. Successful economic growth has meant that most citizens of the United States today live better, along almost every dimension of material life, than did even the rich elite in pre-industrial times.

It is definitely possible for good and bad policies to accelerate or retard long-run economic growth. Argentines were richer than Swedes in the years back before World War I began in 1914, but Swedes today have perhaps four times the material standard of living and the economic productivity level of Argentines. Almost all of this difference is due to differences in growth policies--good policies in the case of Sweden, bad policies in the case of Argentina—for there were few important differences in initial conditions at the start of the twentieth century to give Sweden an edge.

Policies and initial conditions work to accelerate or retard growth through two principal channels. The first is their impact on the available level of *technology* that multiplies the efficiency of labor. The second is their impact on the *capital intensity* of the economy—the stock of machines, equipment, and buildings which the average worker has at his or her disposal.

Better Technology

The overwhelming part of the answer to the question of why Americans today are more productive and better off than their predecessors of a century or two ago is “better technology." We now know how to make electric motors, dope semiconductors, transmit signals over fiber optics, fly jet airplanes, machine internal combustion engines, build tall and durable structures out of concrete and steel, record entertainment programs on magnetic tape, make hybrid seeds, fertilize crops with better mixtures of nutrients, organize an assembly line for mass production, and a host of other things that our predecessors did not know how to do a century or so ago. Perhaps more important, the American economy is equipped to make use of all of these technological discoveries.

Better technology leads to a higher level of the *efficiency of labor*--the skills and education of the labor force, the ability of the labor force to handle modern machine technologies, and the efficiency with which the economy's businesses and markets function. An economy in which the efficiency of labor is higher will be a richer and a more productive economy. This technology-driven overwhelming increase in the efficiency with which we work today is the major component of our current relative prosperity.

Thus it is somewhat awkward to admit that economists know relatively little about better technology. Economists are good at analyzing the consequences of better technology. But they have less to say than they should about the sources of better technology. (We shall return to what economists do have to say about the sources of better technology toward the end of chapter 5.)

Capital Intensity

The second major factor determining the prosperity and growth of an economy—and the second channel through which changes in economic policies can affect long-run growth—is the *capital intensity* of the economy. How much does the average worker have at his or her disposal in the way of capital goods--buildings, freeways, docks, cranes, dynamos, numerically-controlled machine tools, computers, molders, and all the others? The larger the answer to this question, the more prosperous an economy will be: a more capital-intensive economy will be a richer and a more productive economy.

There are, in turn, two principal determinants of capital intensity. The first is the *investment effort* being made in the economy: the share of total production--real GDP--saved and invested in order to increase the capital stock of machines, buildings, infrastructure, and other human-made tools that amplify the productivity of workers. Policies that create a higher level of investment effort lead to a faster rate of long-run economic growth.

The second determinant are the *investment requirements* of the economy: how much of new investment goes to simply equip new workers with the economy's standard level of capital, or to replace worn-out and obsolete machines and buildings. The ratio between the investment effort and the investment requirements of the economy determines the economy's capital intensity.

This chapter focuses on building the intellectual tools that economists have to analyze long-run growth. It outlines a relatively simple framework for thinking about the key growth issues. This chapter looks at the theory. The following chapter looks at the facts of economic growth.

Note that—as mentioned above—the tools have relatively little to say about the determinants of technological progress. They do, however, have a lot to say about the determinants of the capital intensity of the economy. And they have a lot to say about how evolving technology and the determinants of capital intensity together shape the economy's long-run growth.

4.2 The Standard Growth Model

Economists begin to analyze long-run growth by building a simple, standard model of economic growth—a *growth model.* This standard model is also called the Solow model, after Nobel Prize-winning M.I.T. economist Robert Solow. The second thing economists do is to use the model to look for an *equilibrium*--a point of balance, a condition of rest, a state of the system toward which the model will converge over time. Once you have found the equilibrium position toward which the economy tends to move, you can use it to understand how the model will behave. If you have built the right model, this will tell you in broad strokes how the economy will behave.

In economic growth economists look for the *steady-state balanced-growth equilibriu*m. In a steady-state balanced-growth equilibrium the capital intensity of the economy is stable. The economy's capital stock and its level of real GDP are growing at the same proportional rate. And the capital-output ratio--the ratio of the economy's capital stock to annual real GDP--is constant.

The Production Function

The first component of the model is a *behavioral relationship* called the *production function.* This behavioral relationship tells us how the productive resources of the economy—the labor force, the capital stock, and the level of technology that determines the efficiency of labor—can be used to produce and determine the level of output in the economy. The total volume of production of the goods and services that consumers, investing businesses, and the government wish for is limited by the available resources. The production function tells us how available resources limit production.

Tell the production function what resources the economy has available, and it will tell you how much the economy can produce. Abstractly, we write the production function as:

$$(Y/L) = F((K/L), E)$$

This says that real GDP per worker (Y/L)--real GDP Y divided by the number of workers L—is systematically related, in a pattern prescribed by the form of the function F(), to the economy's available resources: the capital stock per worker (K/L), and the current efficiency of labor (E) determined by the current level of technology and the efficiency of business and market organization.

The Cobb-Douglas Production Function

As long as the production function is kept at the abstract level of an F()—one capital letter and two parentheses—it is not of much use. We know that there is a relationship between resources and production, but we don't know what that relationship is. So to make things less abstract—and more useful--henceforth we will use one particular form of the production function. We will use the so-called Cobb-Douglas production function, a functional form that economists use because it makes many kinds of calculations relatively simple. The Cobb-Douglas production function staistes:

$$(Y/L) = (K/L)^{\alpha} \times (E)^{1-\alpha}$$

The economy's level of output per worker (Y/L) is equal to the capital stock per worker K/L raised to the exponential power of some number α, and then multiplied by the current efficiency of labor E raised to the exponential power (1- α).

Figure 4.1: The Cobb-Douglas Production Function, for Parameter α Near Zero

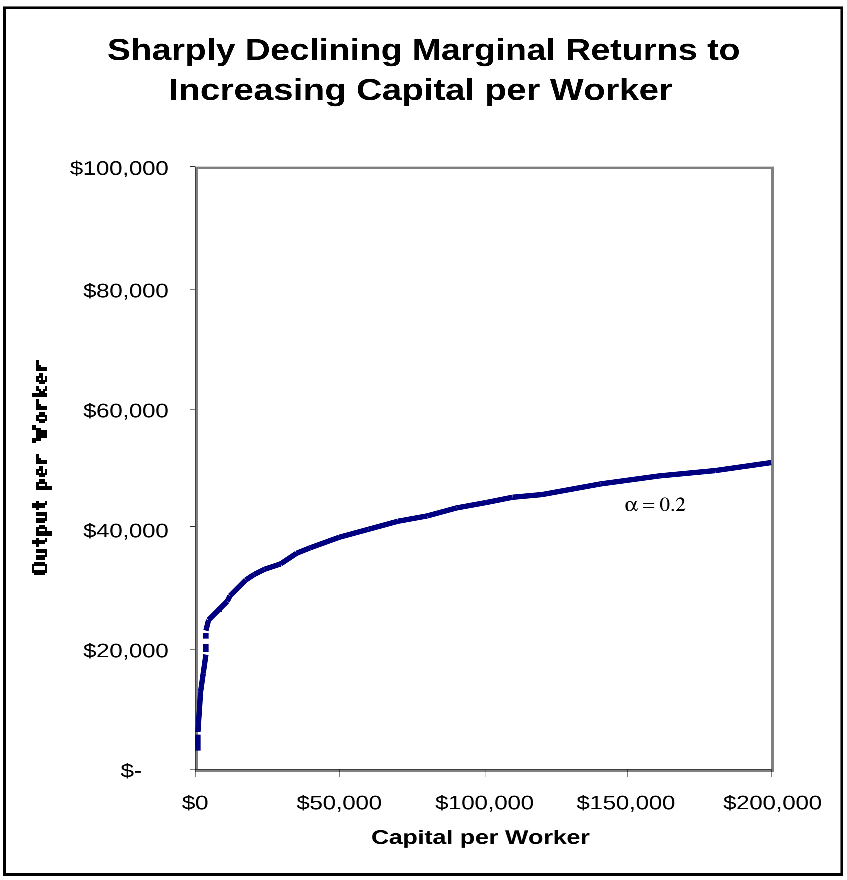


Figure Legend: When the parameter α is close to zero, an increase in capital per worker produces much less in increased output than the last increase in capital per worker. Diminishing returns to capital accumulation set in rapidly and ferociously.

The efficiency of labor E and the number α are *parameters* of the model. The parameter α is always a number between zero and one. The best way to think of it as the parameter that governs how fast diminishing returns to investment set in. A level of α near zero means that the extra amount of output made possible by each additional unit of capital declines very quickly as the capital stock increases, as Figure 4.1 shows.

Figure 4.2: The Cobb-Douglas Production Function, for Parameter α Near One

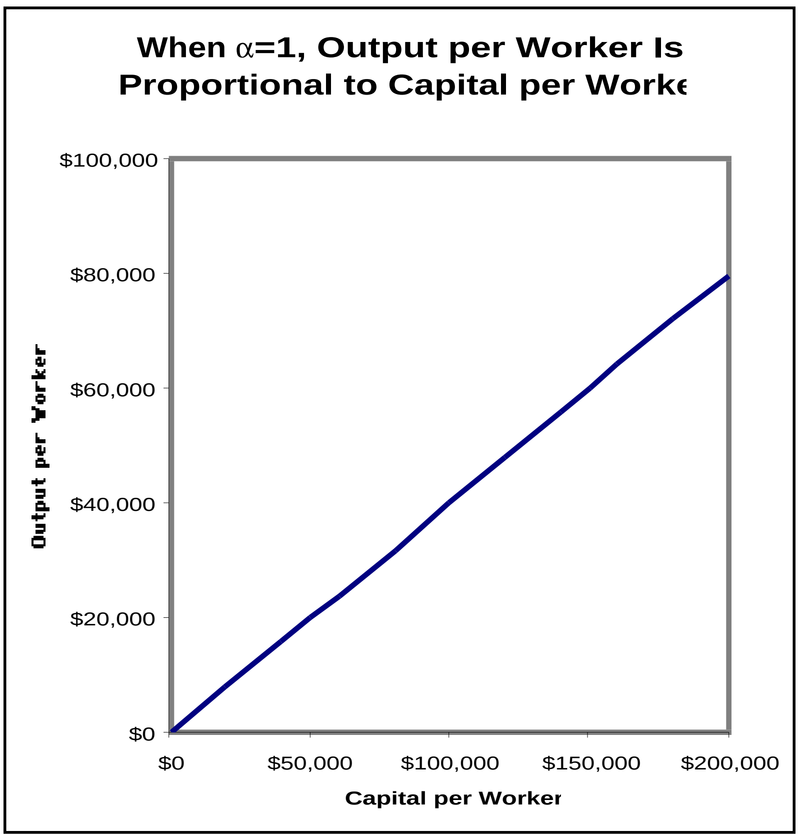


Figure Legend: When α=1, doubling capital per worker doubles output per worker.

There are no diminishing returns to capital accumulation. When the parameter α is near to but less than one, diminishing returns to capital accumulation set in slowly and gently.

By contrast, a level of α near one means that the next additional unit of capital makes possible almost as large an increase in output as the last additional unit of capital, as Figure 4.2 shows. When α equals one, output is proportional to capital: double the stock of capital per worker, and you double output per worker as well. When the parameter α is near to but less than one, diminishing returns to capital accumulation do set in, but they do not set in rapidly or steeply. And as α varies from a high number near one to a low number near zero, the force of diminishing returns gets stronger.

Figure 4.3: The Cobb-Douglas Production Function Is Flexible

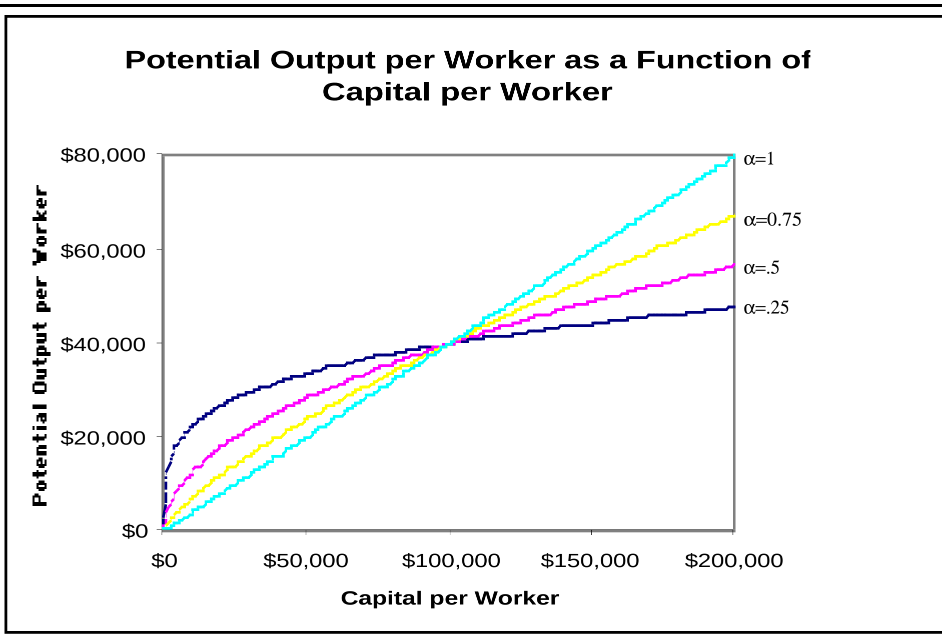


Figure Legend: By changing α--the exponent of the capital-labor ratio

(K/L) in the algebraic form of the production function--you change the curvature of the production function, and thus the point at which diminishing returns to further increases in capital per worker set in. Raising the parameter α increases the speed with which the returns to increased capital accumulation diminish.

The other parameter E tells us the current level of the efficiency of labor. A higher level of E means that more output per worker can be produced for each possible value of the capital stock per worker. A lower value of E means that the economy is very unproductive: not even huge amounts of capital per worker will boost output per worker to achieve what we would think of as prosperity. Box 4.1 illustrates how to use the production function once you know its form and parameters--how to calculate output per worker once you know the capital stock per worker.

The Cobb-Douglas production function is "flexible" in the sense that it can be tuned to fit any of a wide variety of different economic situations. Figure 4.3 shows a small part of the flexibility of the Cobb-Douglas production function. Is the level of productivity high? The Cobb-Douglas function can fit with a high initial level of the efficiency of labor E. Does the economy rapidly hit a wall as capital accumulation proceeds and find that all the investment in the world is doing little to raise the level of production? Then the Cobb-Douglas function can fit with a low level--near zero--of the diminishing-retuns-to-capital parameter α. Is the speed with which diminishing-returns-to-investment sets in moderate? Then pick a moderate value of α, and the Cobb-Douglas function will once again fit.

No economist believes that there is, buried somewhere in the earth, a big machine that forces the level of output per worker to behave exactly as calculated by the algebraic production function above. Instead, economists think that the Cobb-Douglas production function above is a simple and useful approximation.

The true process that does determine the level of output per worker is an immensely complicated one: everyone in the economy is part of it. And it is too complicated to work with. Writing down the Cobb-Douglas production function is a breathtakingly large leap of abstraction. Yet it is a useful leap, for this approximation is good enough that using it to analyze the economy will get us to approximately correct conclusions.

Box 4.1-- Using the Production Function: An Example

For given values of E (say 10000) and α (say 0.3), this production function tells us how the capital stock per worker is related to output per worker. If the capital stock per worker were $250,000, then output per worker would be:

$$Y/L = (\$250000)^{0.3} \times (10000)^{0.7}$$
$$Y/L = \$41.628 \times 630.958$$
$$Y/L = \$26{,}265$$

And if the capital stock per worker were $125,000, then output per worker would be:

$$Y/L = (\$125000)^{0.3} \times (10000)^{0.7}$$
$$Y/L = \$33.812 \times 630.958$$
$$Y/L = \$21{,}334$$

Note that the first $125,000 of capital boosted production from $0 to $21,334, and that the second $125,00 of capital boosted production from $21,334 to $26,265: less than a quarter as much. These substantial diminishing returns should not be a surprise: the value of α in this example--0.3--is low, and low values are supposed to produce rapidly diminishing returns to capital accumulation.

Now nobody expects anyone to raise \$250,000 to the 0.3 power in their head and come up with 41.628. That is what calculators are for. This Cobb-Douglas form of the production function with its fractional exponents carries the drawback that we cannot expect students (or professors!) to do problems in their heads or with just pencil-and-paper. However, this Cobb-Douglas form of the production function also carries substantial benefits: by varying just two numbers--the efficiency of labor E and the diminishing-returns-to-capital parameter α--we can consider and analyze a very broad set of relationships between resources and the economy's productive power.

In fact, this particular Cobb-Douglas form for the production function with all these αs and $(1-\alpha)$s as exponents was built by Cobb and Douglas for precisely for this purpose: so that it would be simple to, by judicious choice of different values of E and α, "tune" the function so that it could capture a large range of different kinds of behavior..

The Rest of the Growth Model

The rest of the growth model is straightforward. First comes the need to keep track of the quantities of the model over time. Do so by attaching to each variable--like the capital stock or the efficiency of labor or output per worker or the labor force--a little subscript telling what year it applies to. Thus K_{1999} will be the capital stock in year 1999. If we want to refer to the efficiency of labor in the current year (but don't much care what the current year is), we will use a *t* (for "time) as a placeholder to stand in for the numerical value of the current year. Thus we write: E_t. And if we want to refer to the efficiency of labor in the year after the current year, we will write: E_{t+1}.

Population Growth

Second comes the pattern of labor force growth. We assume—once again making a simplifying leap of abstraction--that the labor force L of the economy is growing at a constant proportional rate given by the value of a parameter *n*. Note that n does not have to be the same across countries, and can shift over time in any one country). Thus between this year and the next the labor force will grow so that:

$$L_{t+1} = (1 + n) \times L_t$$

Next year's labor force will be n percent higher than this year's labor force, as Figure 4.4 shows.

Figure 4.4: Constant Labor Force Growth

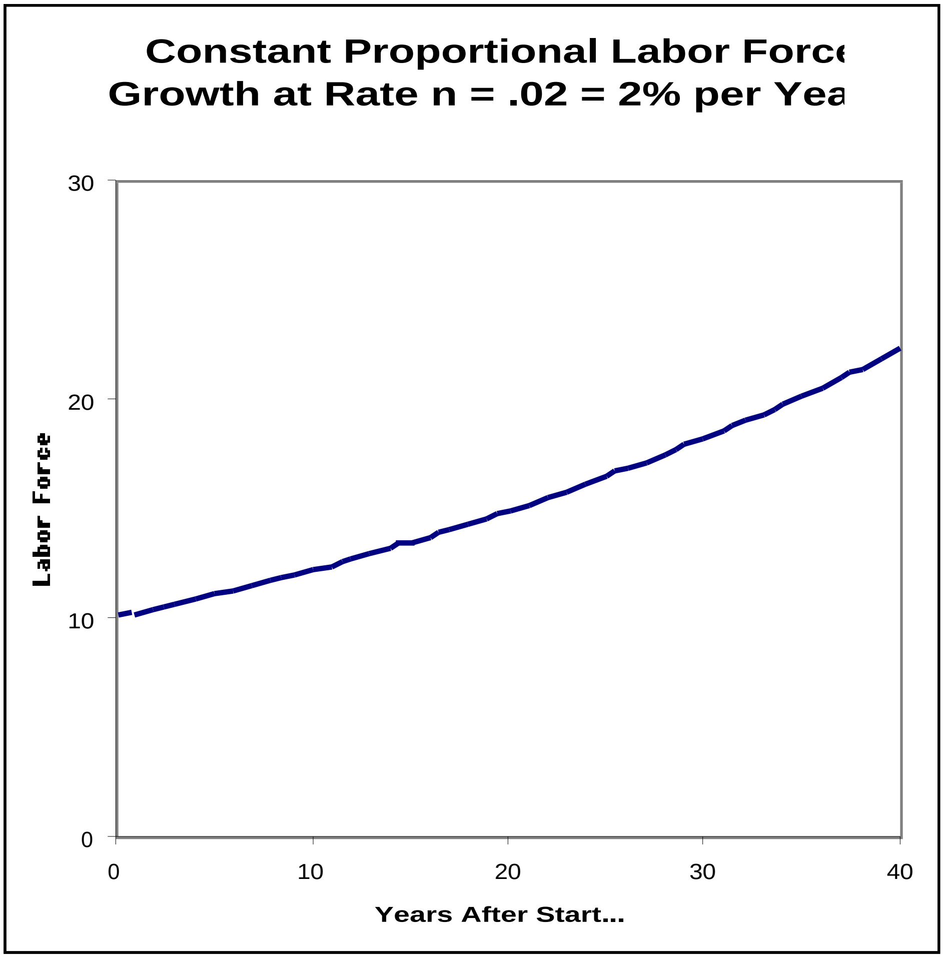


Figure Legend: A labor force increasing at a rate of 2% per year will double roughly every 35 years.

Thus if this year's labor force were 10 million, and the growth rate parameter n were 2 percent per year, then next year's labor force would be:

$$L_{t+1} = (1+n) \times L_t$$
$$L_{t+1} = (1+2\%) \times L_t$$
$$L_{t+1} = (1+0.02) \times 10$$
$$L_{t+1} = 10.2 \;\; million$$

We assume that the rate of growth of the labor force is constant not because we believe that labor force growth is unchanging, but because it makes the analysis of the model simpler. This tradeoff between realism in the model's description of the world and simplicity as a way to make the model easier to analyze is one that economists always face. They usually resolve it in favor of simplicity.

Efficiency of Labor

Assume, also, that the efficiency of labor E is growing at a constant proportional rate given by a parameter g. (Note that g does not have to be the same across countries, and can shift over time in any one country.) Thus between this year and the next year:

$$E_{t+1} = (1+g) \times E_t$$

Next year's level of the efficiency of labor will be g percent higher than this year's level, as Figure 5 shows. Thus if this year's efficiency of labor were \$10,000 per year, and the growth rate parameter g were 1.5 percent per year, then next year the efficiency of labor would be:

$$E_{t+1} = (1+g) \times E_t$$
$$E_{t+1} = (1+0.015) \times \$10,000$$
$$E_{t+1} = \$10,150$$

Figure 4.5: Constant Growth in the Efficiency of Labor

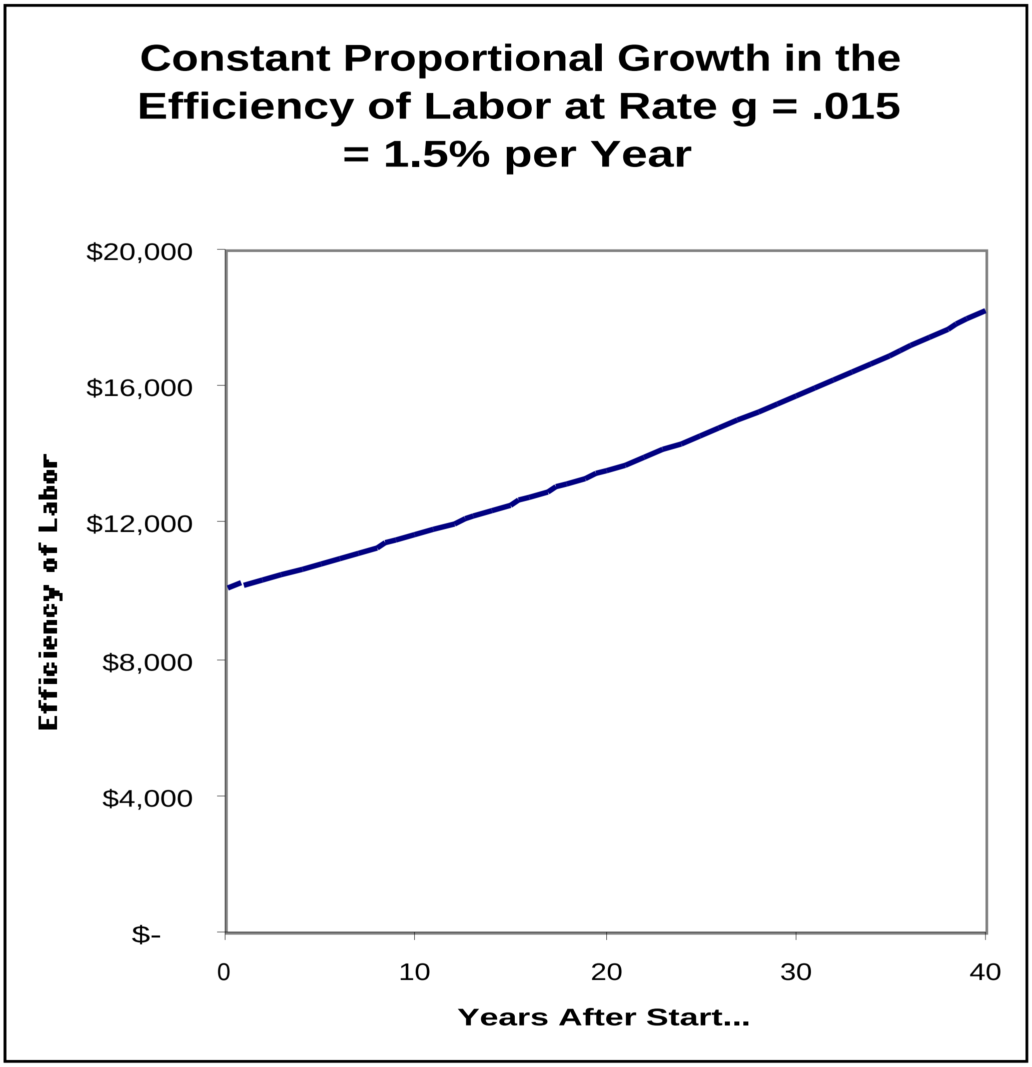


Figure Legend: If the efficiency of labor grows at a constant proportional rate of 1.5 percent per year, it will take about 47 years for it to double.

Once again this assumption is made because it makes the analysis of the model easier, not because the rate at which the efficiency of labor grows is constant.

Savings and Investment

Last, assume that a constant proportional share, equal to a parameter s, of real GDP is saved each year and invested. These gross investments add to the capital stock, so a higher amount of savings and investment means faster growth for the capital stock.

Figure 4.6: Changes in the Capital Stock

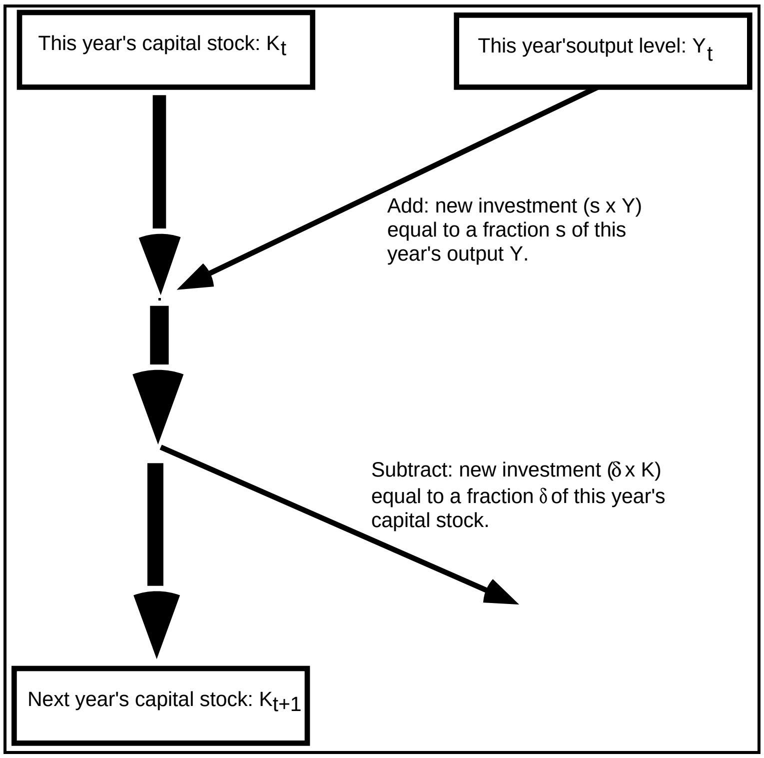


Figure Legend: Gross investment adds to and depreciation subtracts from the capital

stock. Depreciation is a share δ of the current capital stock. Investment is a share s of current production.

But the capital stock does not grow by the full amount of *gross* investment. A fraction δ (the Greek letter lower-case delta, for depreciation) of the capital stock wears out or is scrapped each period. Thus the actual relationship between the capital stock now and the capital stock next year is:

$$K_{t+1} = K_t + (s \times Y_t) - (\delta \times K_t)$$

The level of the capital stock next year will be equal to the capital stock this year, plus the savings rate s times this year's level of real GDP, minus the depreciation rate δ times this year's capital stock, as Figure 4.6 shows. Box 4.2 illustrates how to use this capital accumulation equation to calculate the capital stock.

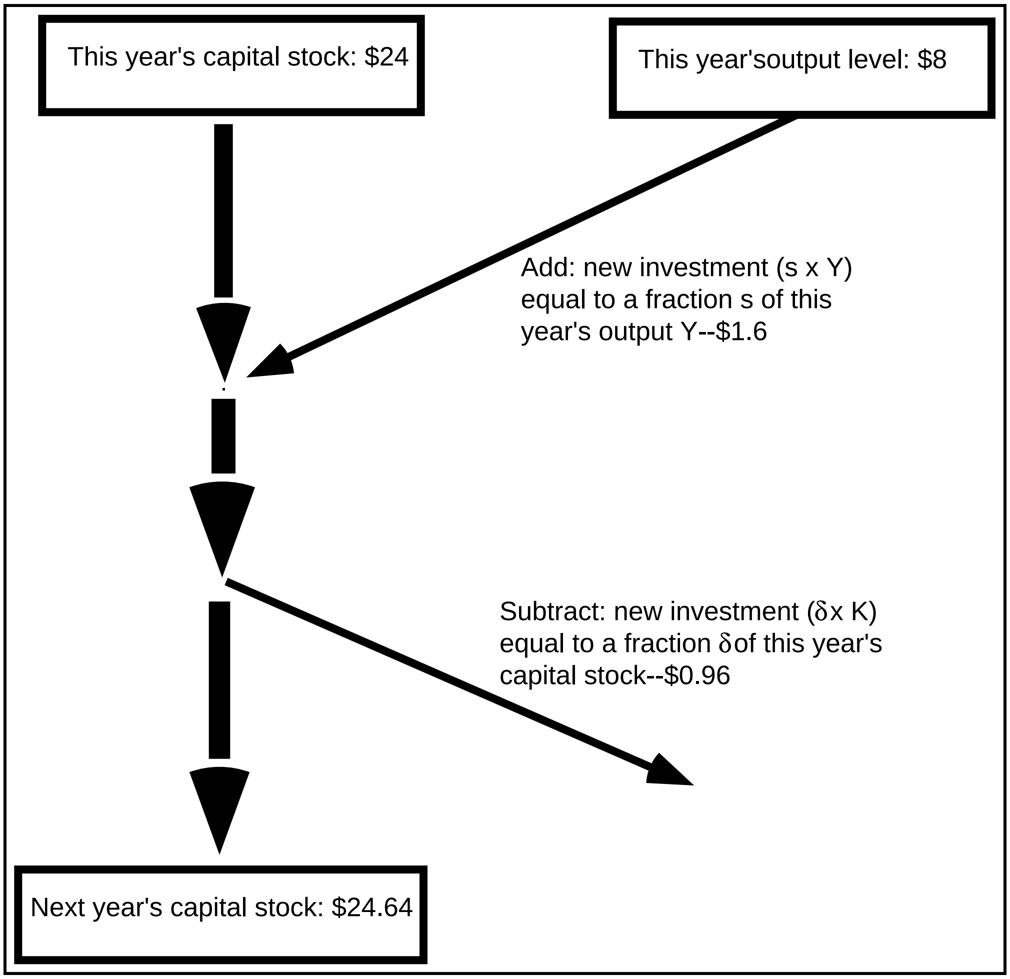
Box 4.2-- Investment, Depreciation, and Capital Accumulation: An Example

For example, suppose that the current level of output in the economy is $8 trillion a year and the current year's capital stock in the economy is $24 trillion. Then a savings rate s of 20 percent and an annual depreciation rate δ of 4 percent would mean that next year's capital stock will be:

$$K_{t+1} = K_t + (s \times Y_t) - (\delta \times K_t)$$
$$K_{t+1} = \$24 + (0.2 \times \$8) - (0.04 \times \$24)$$
$$K_{t+1} = \$24 + \$1.6 - \$0.96$$
$$K_{t+1} = \$24.64 \quad \textit{trillion}$$

Between this year and next year the capital stock has grown by $640 billion. That is a proportional growth rate of 2.667%.

Additions to and Subtractions from the Capital Stock:



That is all there is to the growth model: three assumptions about rates of population growth, increases in the efficiency of labor, and investment, plus one additional equation to describe how the capital stock grows over time. Those plus the production function make up the growth model. It is simple. But understanding the processes of economic growth that the model generates is more complicated.

4.3 Understanding the Growth Model

Economists' first instinct when analyzing any model is to look for a point of *equilibrium.* They look for a situation in which the quantities and prices being analyzed are stable and unchanging. And they look for the economic forces to push an out-of-equilibrium economy to one of its points of equilibrium. Thus microeconomists talk about the equilibrium of a particular market. Macroeconomists talk (and we will talk later on in the book) about the equilibrium value of real GDP relative to potential output.

In the study of long-run growth, however, the key economic quantities are never stable. They are growing over time. The efficiency of labor is growing, the level of output per worker is growing, the capital stock is growing, the labor force is growing. How, then, can we talk about a point of equilibrium where things are stable if everything is growing?

The answer is to look for an equilibrium in which everything is growing together, at the same proportional rate. Such an equilibrium is one of *steady-state balanced growth.* If everything is growing together, then the relationships between key quantities in the economy are stable. And it makes this chapter easier if we focus on one key ratio: the capital-output ratio. Thus our point of equilibrium will be one in which the capital-output ratio is constant over time, and toward which the capital-output ratio will converge if it should find itself out of equilibrium.

How Fast Is the Economy Growing?

So how fast are the key quantities in the economy growing? We know that they are growing. The efficiency of labor is, after all, increasing at a proportional rate g. And we know that it is technology-driven improvements in the efficiency of labor that have generated most of the increases in our material welfare and economic productivity over the past few centuries.

Determining how fast the quantities in the economy are growing is straightforward if we remember our three mathematical rules:

- The proportional growth rate of a product --P x Q, say--is equal to the sum of the proportional growth rates of the factors, is equal to the growth rate of P plus the growth rate of Q.

- The proportional growth rate of a quotient--E/Q, say--is equal to the difference of the proportional growth rates of the dividend (E) and the divisor (Q).

- The proportional growth rate of a quantity raised to a exponent--Q^y, say--is equal to the exponent (y) times the growth rate of the quantity (Q).

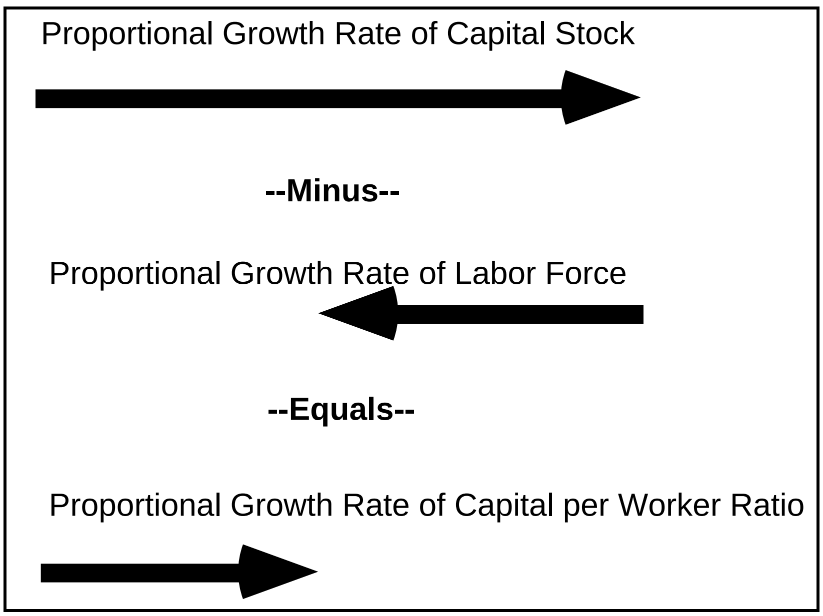
The Growth of Capital per Worker

Begin with capital per worker. To save on our breath and reduce the length of equations, let's use the expression $g(k_t)$ to stand for the proportional growth rate of capital per worker. The proportional growth rate is simply what output per worker will be next year minus what output per worker is this year, all divided by what output per worker is this year:

$$g(k_t) = \frac{(K_{t+1}/L_{t+1}) - (K_t/L_t)}{(K_t/L_t)}$$

Capital-per-worker is a quotient: it is the capital stock divided by the labor force. Thus the proportional growth rate of capital-per-worker is the growth rate of the capital stock minus the growth rate of the labor force, as Figure 4.7 shows.

Figure 4.7: Calculating the Proportional Growth Rate of the Capital per Worker Ratio



The growth rate of the labor force is simply the parameter n. That's what the parameter n is. The growth rate of the capital stock is a bit harder to calculate. We know that it is:

$$\frac{K_{t+1} - K_t}{K_t}$$

And we know that we can write next year's capital stock as equal to this year's capital stock, plus gross investment, minus depreciation:

$$K_{t+1} = \left(K_t + (s \times Y_t) - (\delta \times K_t)\right)$$

If we substitute in for next year's capital stock, and rearrange:

$$\frac{\left(K_t + (s \times Y_t) - (\delta \times K_t)\right) - K_t}{K_t} = \frac{s \times Y_t}{K_t} - \delta \frac{K_t}{K_t} = \frac{s \times Y_t}{K_t} - \delta$$

Then we see that the proportional growth rate of capital per worker is:

$$g(k_t) = \frac{s}{\left(K_t / Y_t\right)} - \delta - n$$

To make our equations look simpler, let's give the capital-output ratio K/Y a special symbol: κ--a little k with a short stem (actually the Greek letter kappa)--and write that the proportional growth rate of capital per worker is:

$$g(k_t) = s / \kappa_t - \delta - n$$

This says that the growth rate of capital-per-worker is equal to the savings share of GDP (s) divided by the capital-output ratio (κ), minus the depreciation rate (δ), minus the labor force growth rate (n). Box 4.3 goes through example calculations of what the growth rate of capital-per-worker is for sample parameter values. A higher rate of labor force growth will reduce the rate of growth of capital per worker: more workers means the available capital has to be divided up more ways. A higher rate of depreciation will reduce the rate of growth of capital per worker: more capital will rust away. And a higher capital-output ratio will reduce the proportional growth rate of capital per worker: the higher the capital-output ratio, the smaller is investment relative to the capital stock.

Box 4.3--The Growth of Capital per Worker: An Example

Suppose that the proportional growth rate of the labor force n were 2 percent per year--0.02. Suppose also that the depreciation rate δ were 4 percent per year and the savings rate were 20 percent. We can then calculate what the proportional rate of growth of capital per worker would be for each possible level of the capital-output ratio. Simply substitute the values of depreciation, labor force growth, and the savings rate into the equation for the growth rate of capital per worker:

$$g(k_t) = s/\kappa_t - \delta - n$$

to get:

$$g(k_t) = .20/\kappa_t - 0.04 - 0.02$$

Then if the current capital-output ratio were to be five, the growth rate of capital per worker would be:

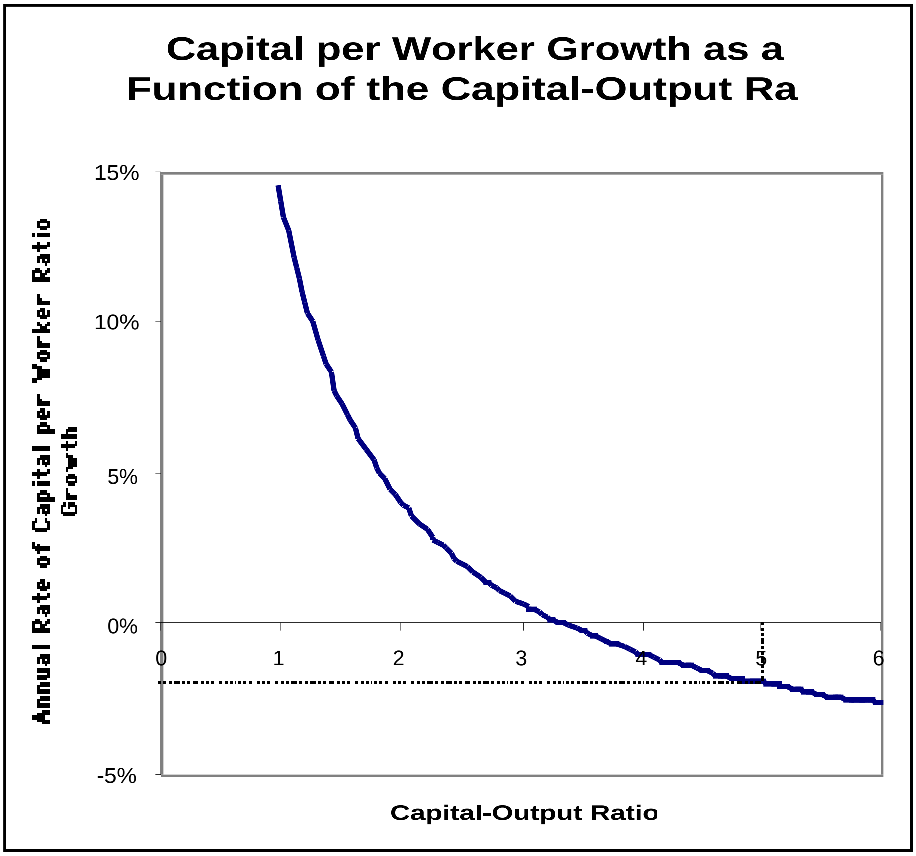
$$g(k_t) = \frac{0.20}{5} - 0.04 - 0.02 = .04 - .04 - .02 = -.02$$

minus 2 percent per year: the capital per worker ratio would be shrinking. By contrast ,if the current capital-output ratio were to be 2.5, the growth rate of capital per worker would be:

$$g(k_t) = \frac{0.20}{2.5} - 0.04 - 0.02 = .08 - .04 - .02 = +.02$$

plus 2 percent per year: the capital per worker ratio would be growing.

Capital per Worker Growth



Legend: The growth rate of capital per worker plotted as a function of the capital-output ratio for the following parameter values: labor force growth rate n of 0.02, a depreciation rate δ of 0.04, and a savings rate s of 0.20. The higher the capital-output ratio, the lower is the growth rate of capital per worker.

The Growth of Output per Worker

Our Cobb-Douglas form of the production function tells us that the level of output per worker is:

$$(Y_t / L_t) = (K_t / L_t)^{\alpha} \times (E_t)^{1-\alpha}$$

Output per worker is the product of two terms, each of which is a quantity raised to a exponential power. So using our mathematical rules of thumb the proportional growth rate of output per worker--call it $g(y_t)$ to once again save on space--will be, as Figure 4.8 shows:

- α times the proportional growth rate of capital per worker,
- plus (1 - α) times the rate of growth of the efficiency of labor.

Figure 4.8: Calculating the Growth Rate of Output per Worker

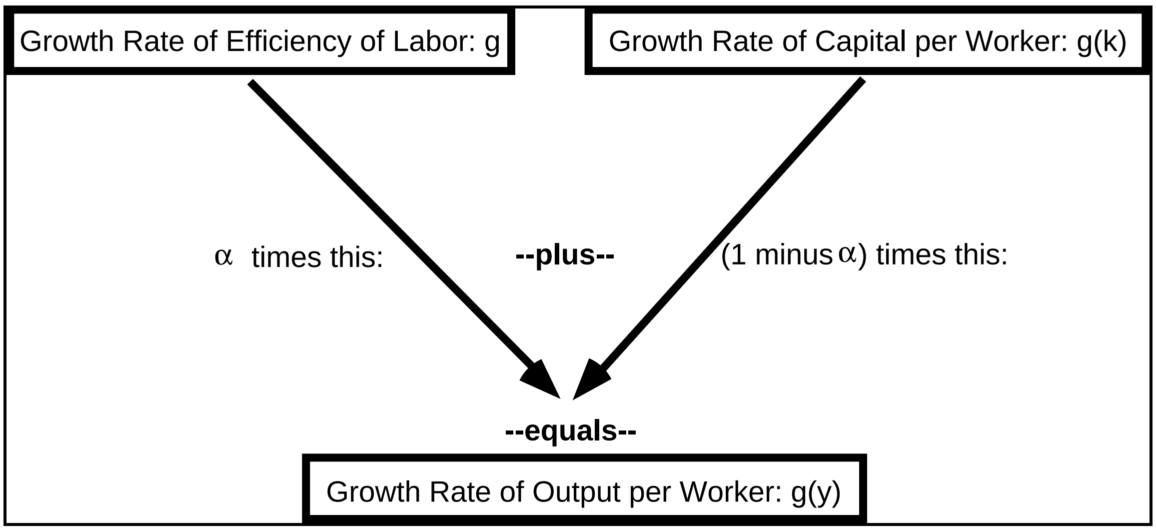


Figure Legend: The growth rate of output per worker is a weighted average of the growth rates of capital per worker and of the efficiency of labor.

The rate of growth of the efficiency of labor is simply g. And the previous section calculated the growth rate of capital per worker g(k): $s/\kappa_t - \delta - n$.

So simply plug these expressions in:

$$g(y_t) = \left[\alpha \times \left\{s/\kappa_t - \delta - n\right\}\right] + [(1-\alpha) \times g]$$

And simplify a bit by rearranging terms:

$$g(y_t) = g + \left[\alpha \times \left\{ s/\kappa_t - (n + g + \delta) \right\} \right]$$

Box 4.4 shows how to calculate the growth rate of output per worker in a concrete case.

Box 4.4--The Growth of Output per Worker: An Example

For example, suppose that we are analyzing an economy in which the growth rate of the efficiency of labor g is 0.02, the diminishing-returns-to-investment parameter α is 0.5, the labor force growth rate n is, 0.02, the depreciation rate δ 0.04, and the savings rate s 0.3. Then we can determine the current proportional rate of growth of output per worker by substituting the values of the parameters into the equation:

$$g(y_t) = g + \left[\alpha \times \left\{ s/\kappa_t - (n + g + \delta) \right\} \right]$$

to get:

$$g(y_t) = 0.02 + \left[0.5 \times \left\{ 0.3/\kappa_t - (0.02 + 0.02 + 0.04) \right\} \right]$$

If the capital-output ratio were 3, then the proportional rate of growth of output per worker would be:

$$g(y_t) = 0.02 + \left[0.5 \times \left\{ \frac{0.3}{3} - (0.02 + 0.02 + 0.04) \right\} \right] = .02 + .5 \times .02 = .03$$

It would be 3% per year.

If the capital-output ratio were 6, then the proportional rate of growth of output per worker would be:

$$g(y_t) = 0.02 + \left[0.5 \times \left\{ \frac{0.3}{6} - (0.02 + 0.02 + 0.04) \right\} \right] = .02 + .5 \times -.03 = .005$$

It would be half a percent per year.

The Growth of the Capital-Output Ratio

Now consider the capital-output ratio κ_t. It will be the key ratio that we will focus on--and our equilibrium will be when it is stable and constant. The capital-output ratio is equal to capital per worker divided by output per worker. So its proportional growth rate is the difference between their growth rates:

$$g(\kappa_t) = g(k_t) - g(y_t) = \{s/\kappa_t - \delta - n\} - \{g + \alpha \times \{s/\kappa_t - (n + g + \delta)\}\}$$

Which simplifies to:

$$g(\kappa_t) = (1-\alpha) \times \{s/\kappa_t - (n + g + \delta)\}$$

Thus the growth rate of the capital-output ratio depends on the balance between the *investment requirements*—(n+g+δ)--and the *investment effort*—s--made in the economy. The higher are investment requirements, the lower will be the growth rate of the capital-output ratio, as Figure 4.9 illustrates.

Figure 4.9: Growth of the Capital-Output Ratio

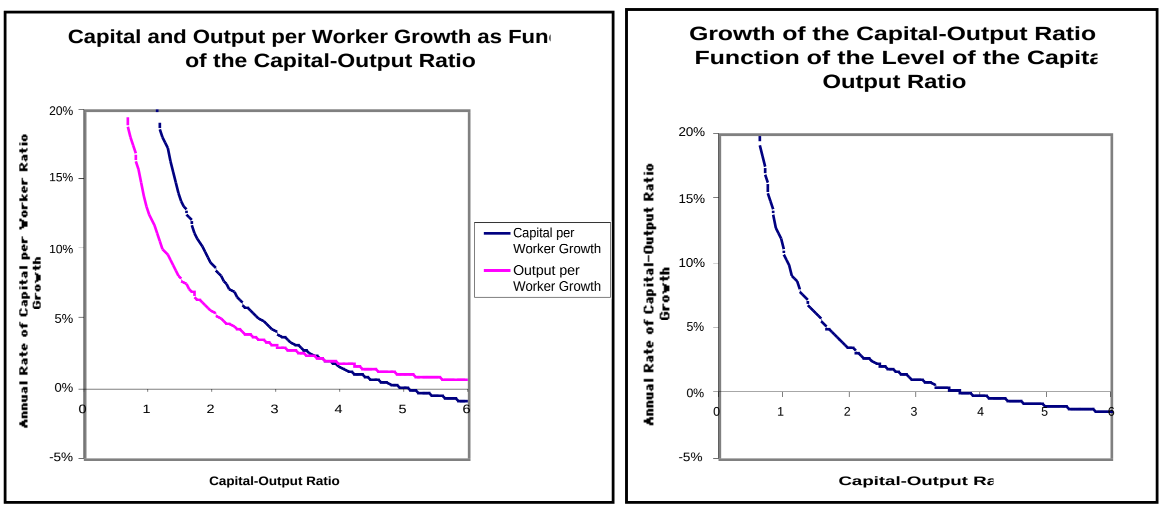


Figure Legend: The proportional growth rates of both capital per worker and output per

worker are decreasing functions of the capital-output ratio. The higher is the capital-output ratio, the slower is growth. The rate of growth of the capital-output ratio itself is also a decreasing function of the capital-output ratio: the gap between capital-per-worker and output-per-worker growth is large and positive when the capital-output ratio is low, and negative when the capital-output ratio is high.

Steady-State Growth Equilibrium

The Capital-Output Ratio

From the growth rate of the capital-output ratio:

$$g(\kappa_t) = (1-\alpha) \times \{ s/\kappa_t - (n+g+\delta) \}$$

We can see that whenever the capital-output ratio κ_t is greater than s/(n+g+δ), the growth rate of the capital-output ratio will be negative. Output per worker will be growing faster than capital per worker. And the capital-output ratio will be shrinking. By contrast, we can also see that whenever the capital-output ratio κ_t is *less* than s/(n+g+δ), the capital-output ratio will be growing. The capital stock per worker will be growing faster than output per worker, as Figure 4.10 shows.

Figure 4.10: Dynamics of the Capital-Output Ratio

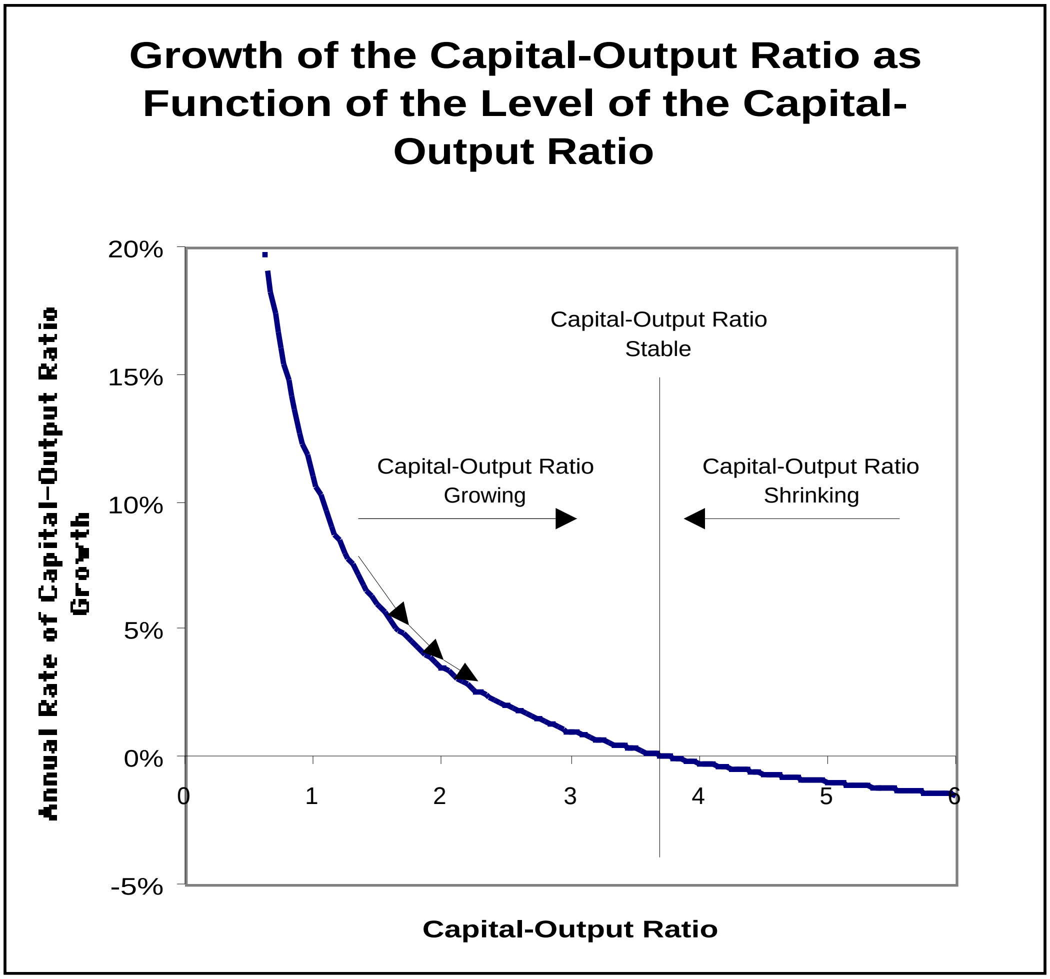


Figure Legend: The value of the capital-output ratio at which its rate of change is zero is an *equilibrium*. If the capital-output ratio is at that *equilibrium* value, it will stay there. If it is away from that equilibrium value, it will head toward it.

What happens when the capital-output ratio κ_t is equal to $s/(n+g+\delta)$? Then the growth rate of the capital-output ratio will be zero. It will be stable, neither growing nor shrinking. If the capital-output ratio is at that value, it will stay there. If the capital-output ratio is away from that value, it will head toward there. No matter where the capital-output ratio κ_t starts, it will head for--converge to--home in on-- its steady-state balanced-growth value of s/(n+g+δ) (see Figure 4.11).

Thus the value s/(n+g+δ) is the *equilibrium level* of the capital-output ratio. It is a point at which the economy tends to balance, and to which the economy converges. The requirement that the capital-output ratio equal this equilibrium level becomes our equilibrium condition for balanced economic growth.

Figure 4.11: Convergence of the Capital-Output Ratio

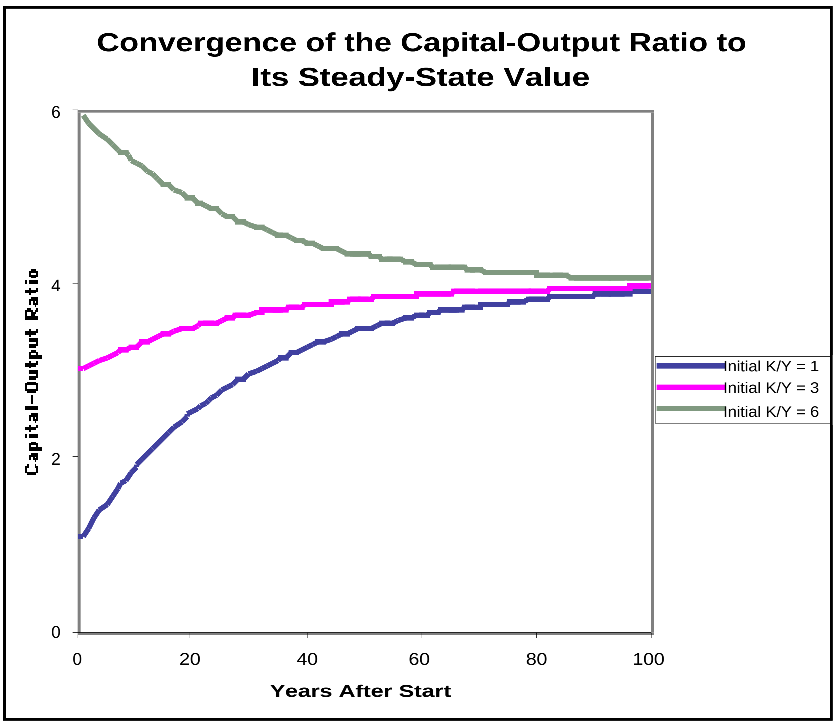


Figure Legend: If the capital-output ratio starts at a value different from its steady-

state equilibrium value, it will head towards equilibrium. The figure shows the paths over time of the capital-output ratio for parameter values of s=0.28, n=0.02, g=0.015, δ=0.035, α=0.5, and for different initial starting values of 1, 3, and 6.

The steady-state capital-output ratio κ^* is 4.

And to make our future equations even simpler, give the quantity $s/(n+g+\delta)$ that is the equilibrium value of the capital-output ratio a special symbol: κ^*:

$$\kappa^* = \frac{s}{n+g+\delta}$$

Other Quantities

When the capital-output ratio κ_t is at its steady state value of:

$$\kappa^* = s/(n+g+\delta),$$

the proportional growth rates of capital per worker and output per worker are stable too. Output per worker is then growing at a proportional rate g:

$$g(y_t) = g$$

The capital stock per worker is then growing at the same proportional rate g:

$$g(k_t) = g$$

The total economy-wide capital stock is then growing at the proportional rate n+g: the growth rate of capital per worker plus the growth rate of the labor force. Real GDP is then also growing at rate n+g: the growth rate of output per worker plus the labor force growth rate.

The Level of Output per Worker On the Steady State Growth Path

When the capital-output ratio is at its steady-state balanced-growth equilibrium value κ*, we say that the economy is on its steady-state growth path. What is the level of output per worker if the economy is on its steady-state growth path? We saw the answer to this this back in chapter 3. The requirement that the economy be on its steady-state growth path was then our equilibrium condition:

$$\left(\frac{K_t}{Y_t}\right) = \kappa^* = \frac{s}{n+g+\delta}$$

And in order to combine it with the production function:

$$\left(Y_t / L_t\right) = \left(K_t / L_t\right)^{\alpha} \times \left(E_t\right)^{1-\alpha}$$

we first rewrote the production function to make capital-per-worker the product of the capital-output ratio and output per worker:

$$\left(Y_t / L_t\right) = \left(Y_t / L_t \times K_t / Y_t\right)^{\alpha} \times \left(E_t\right)^{1-\alpha}$$

Dividing both sides by $(Y/L)^{\alpha}$:

$$\left(Y_t / L_t\right)^{1-\alpha} = \left(K_t / Y_t\right)^{\alpha} \times \left(E_t\right)^{1-\alpha}$$

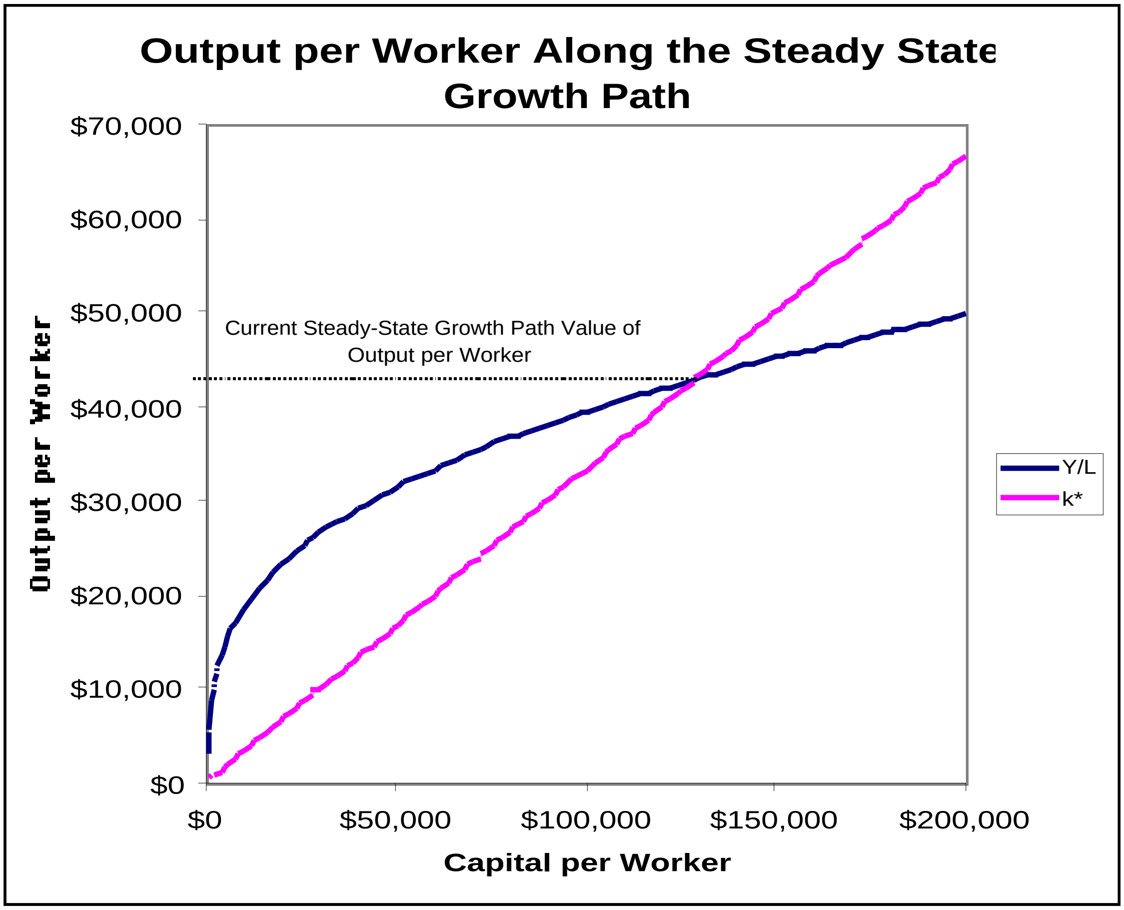
And then raising both sides to the 1/(1-α) power produces an equation for the level of output per worker:

$$\left(Y_t / L_t\right) = \left(K_t / Y_t\right)^{\left(\frac{\alpha}{1-\alpha}\right)} \times \left(E_t\right)$$

Substitute the equilibrium condition into this transformed form of the production function. The result is that, as long as we are on the steady-state balanced-growth path:

$$\left(Y_t / L_t\right) = \left(\frac{s}{n+g+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} \times E_t = \kappa *^{\left(\frac{\alpha}{1-\alpha}\right)} \times E_t$$

Figure 4.12: Calculating Steady-State Output per Worker



Is the algebra too complicated? There is an alternative, diagrammatic way of seeing what the steady-state capital-output ratio implies for the steady-state level of output per worker: Figure 4.12.

Simply draw the production function for the current level of the efficiency of labor E_t. Also draw the line that shows where the capital-output ratio is equal to its steady state value, κ*. Look at the point where the curves intersect. That point shows what the current level of output per worker is along the steady state growth path (for the current level of the efficiency of labor).

Anything that increases the steady-state capital-output ratio will rotate the capital-output line to the right. Thus it will raise steady-state output per worker. Anything that decreases the steady-state capital-output ratio rotates the capital-output line to the left. It thus lowers steady-state output per worker.

If we define:

$$\lambda = \frac{\alpha}{1-\alpha}$$

and call λ the *growth multiplier* (where does the growth multiplier arise from? See Box 4.5), then output per worker along the steady-state growth path is equal to the steady-state capital-output ratio raised to the growth multiplier, times the current level of the efficiency of labor.:

$$\left(\frac{Y_t}{L_t}\right)_{ss} = \kappa^{*\lambda} \times E_t$$

Thus calculating output per worker when the economy is on its steady-state growth path is a simple three-step procedure:

- First, calculate the steady-state capital-output ratio, $\kappa^*=s/(n+g+\delta)$, the savings rate divided by the sum of the population growth rate, the efficiency of labor growth rate, and the depreciation rate.
- Second, amplify the steady-state capital-output ratio κ* by the growth multiplier. Raise it to the $\lambda = (\alpha/(1-\alpha))$ power, where α is the diminishing-returns-to-capital parameter.

- Third, multiply the result by the current value of the efficiency of labor E_t, which can be easily calculated because the efficiency of labor is growing at the constant proportional rate g.

And the fact that an economy converges to its steady-state growth path makes analyzing the long-run growth of an economy relatively easily as well:

- First calculate the steady-state growth path, shown in Figure 4.13.

- From the steady-state growth path, forecast the future of the economy: If the economy is on its steady-state growth path today, it will stay on its steady-state growth path in the future (unless some of the parameters--n, g, δ, s, and α––shift).
- If the economy is not on its steady-state growth path today, it is heading for its steady-state growth path and will get there soon.

Thus long-run economic forecasting becomes simple.

Figure 4.13: Output per Worker On the Steady-State Growth Path

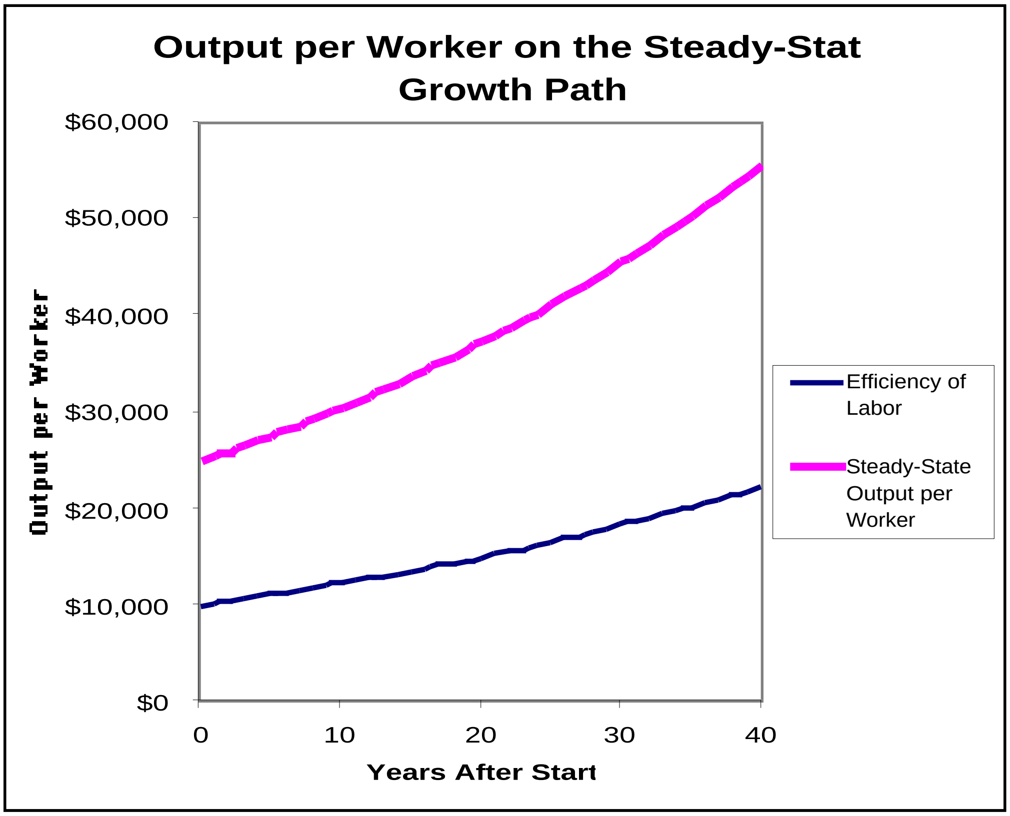


Figure Legend: Parameter values: labor-force growth rate n at 1% per year; increase in the efficiency of labor g at 2% per year; depreciation rate δ at 3% per year; savings rate s at 37.5%; and diminishing-returns-to-capital parameter α at 1/3. The efficiency of labor and output per worker grow smoothly along the economy's balanced growth path.

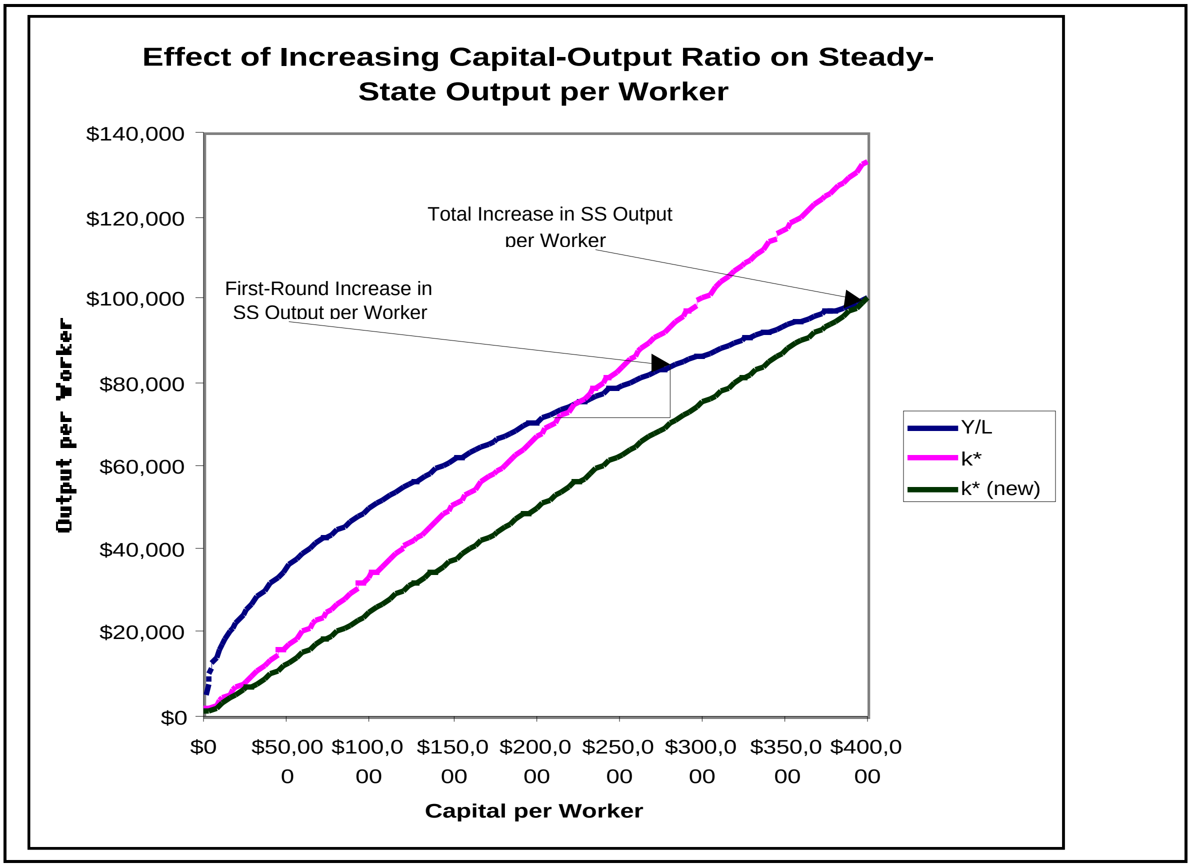
Box 4.5-- Where the Growth Multiplier Comes From: the Details

Why is the steady-state capital-output ratio raised to the (larger) power of (α/(1-α)) rather than just the power α? It makes a big difference when one applies the growth model to different situations.

The reason is that an increase in the capital-output ratio increases the capital stock both directly and indirectly. For the same level of output you have more capital. And because extra output generated by the additional capital is itself a source of additional savings and investment, you have higher capital for that reason as well. The "impact" effect of the additional capital generated by anything that raises κ^*--an increase in savings, or a decrease in labor force, or anything else--is thus multiplied by these positive feedback effects.

The figure below shows the effect of this difference between α and $(\alpha/(1-\alpha))$. An increase in the capital-output ratio means more capital for a given level of output, and that generates the first-round increase in output: amplification by the increase in capital raised to the power α. But the first-round increase in output generates still more capital, which increases production further. The total increase in production is the proportional increase in the steady-state capital-output ratio raised to the (larger) power $(\alpha/(1-\alpha))$.

Figure: The Growth Multiplier



How Fast Does the Economy Head For Its Steady-State Growth Path?

Suppose that the capital-output ratio κ_t is not at its steady state value κ^*? How fast does it approach its steady state? Even in this simple growth model we can't get an exact answer. But if we are willing to settle for approximations, and confine our attention only to small differences beween the current capital-output ratio κ_t and its steady-state value κ^*, then we can get answers. The growth rate of the capital-output ratio will be approximately equal to a fraction $(1-\alpha) \times (n+g+\delta)$ of the gap between the steady-state and its current level.

For example, if (1-α) x (n+g+δ) is equal to 0.04, the capital-output ratio will close approximately 4 percent of the gap between its current level and its steady-state value in a year. If (1-α) x (n+g+δ) is equal to 0.07, the capital-output ratio closes 7 percent of the gap between its current level and its steady-state value in a year. A variable closing 4 percent of the gap each year between its current and its steady-state value will move halfway to its steady-state value in 18 years. A variable closing 7 percent of the gap each year between its current and its steady-state value will move halfway to its steady-state value in 10 years.

Figure 4.14: West German Convergence to Its Steady-State Growth Path

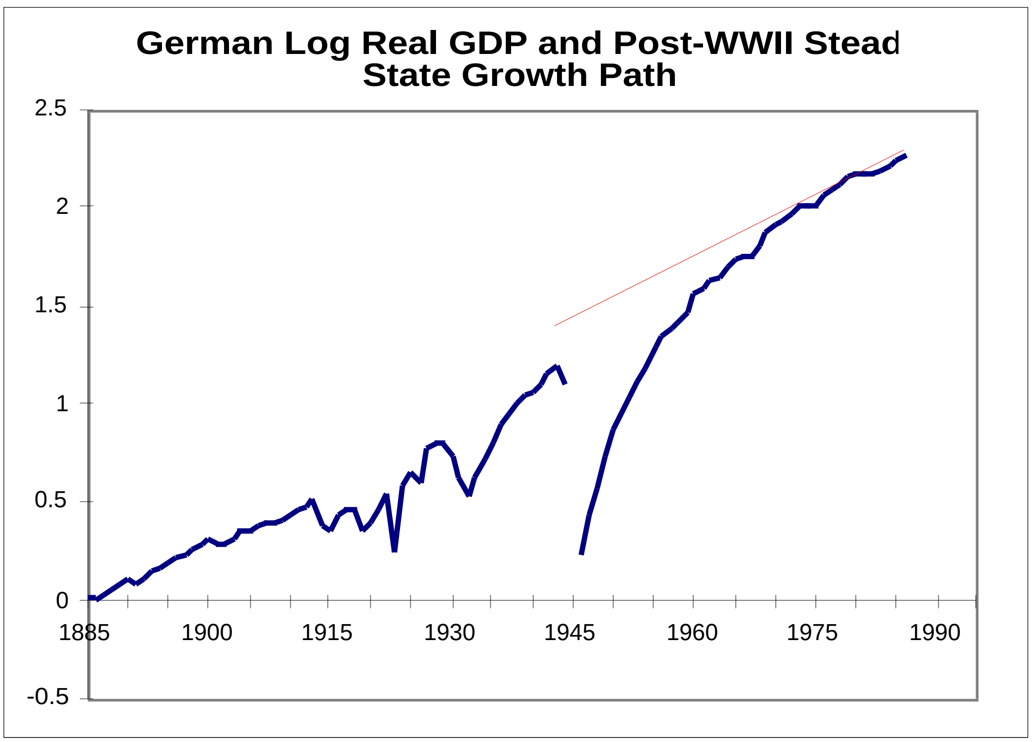


Figure Legend: The end of World War II left the West German economy in ruins. Yet within twelve years it had closed half the gap back to its steady-state growth path. And within thirty years it had closed effectively all of the gap back to its steady-state growth path. Economists study equilibrium steady-state growth paths for a reason: economies do converge to them and then remain on them.

This finding generalizes--as long as we remember that it is only an approximation, and is only even approximately valid for relatively small proportional deviations of the capital-output ratio from its steady-state value. Box 4.6 illustrates how to use this approximation. And this approximation allows us to make much better medium-run forecasts of the dynamic of the economy:

- An economy that is not on its steady-state growth path will close a fraction (1-α) x (n+g+δ) of the gap between its current state and its steady-state growth path in a year.

Box 4.6-- Converging to the Steady-State Balanced-Growth Path: An Example

Thus an economy with parameter values of population growth n=0.02, efficiency of labor growth g=0.015, depreciation δ=0.035, and a diminishing-returns-to-investment parameter α=0.5--the economy whose capital-output ratio is showed in Figure 4.11--would, if off of its steady-state growth path, close a fraction:

$$(1-\alpha)\times(n+g+\delta)=(1-0.5)\times(.02+.015+.035)$$
$$=0.5\times.07=0.035$$

of 3.5 percent of the gap between its current state and its steady-state each year. Such a rate of convergence would allow the economy to close half of the gap to the steady-state in twenty years.

Thus short- and medium-run forecasting becomes simple too. All you have to do is to predict that the economy will head for its steady-state growth path, and calculate what the steady-state growth path is.

Determining the Steady-State Capital-Output Ratio

Labor Force Growth

The faster the growth rate of the labor force, the lower will be the economy's steady-state capital-output ratio. Why? Because each new worker who joins the labor force must be equipped with enough capital to be productive, and to on average match the productivity of his or her peers. The faster the rate of growth of the labor force, the larger the share of current investment that must go to equip new members of the labor force with the capital they need to be productive. Thus the lower will be the amount of investment that can be devoted to building up the average ratio of capital to output.

A sudden and permanent increase in the rate of growth of the labor force will lower the level of output per worker on the steady-state growth path. How large will the long-run change in the level of output be, relative to what would have happened had population growth not increased? It is straightforward to calculate if we know what the other parameter values of the economy are.

How important is all this in the real world? Does a high rate of labor force growth play a role in making countries relatively poor not just in economists' models but in reality? It turns out that it is important, as Figure 4.15 shows. Of the twenty-two countries in the world with GDP per worker levels at least half that of the U.S. level, eighteen have labor force growth rates of less than 2% per year, and twelve have labor force growth rates of less than 1% per year. The additional investment requirements imposed by rapid labor force growth are a powerful reducer of capital intensity, and a powerful obstacle to rapid economic growth.

Figure 4.15: Labor Force Growth and GDP per Worker Levels

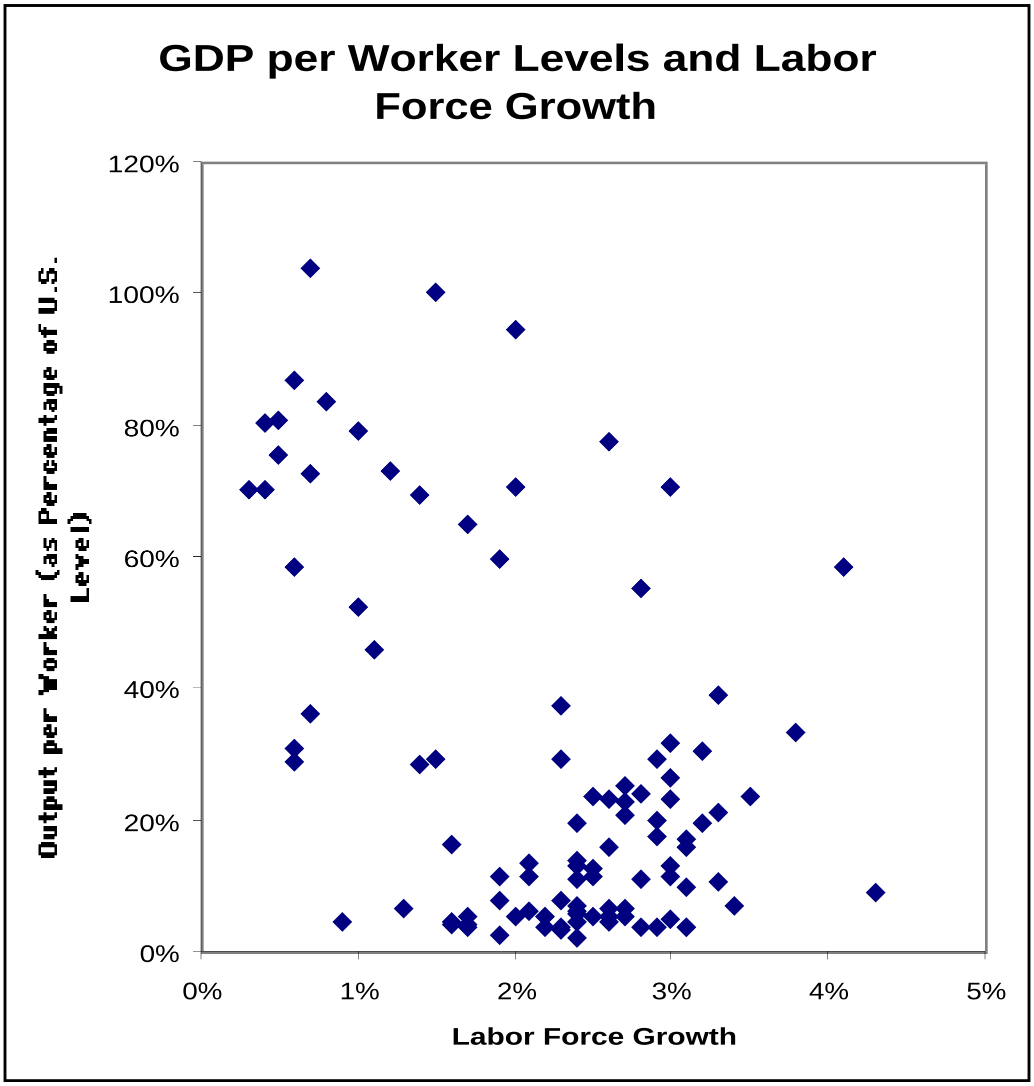


Figure Legend: The average country with a labor force growth rate of less than one percent per year has an output per worker level nearly 60% of the U.S. level. The average country with a labor force growth rate of more than three percent per year has an output per worker level only 20% of the U.S. level. Not all of this is due to a one way relationship from fast population growth to high investment requirements to low steady-state capital-output ratios: countries are not just poor because they have fast labor force growth rates, to some degree they have fast labor force growth rates because they are poor. But some of it is. High labor force growth rates are a powerful cause of relative poverty in the world today.

Source: Author’s calculations from the Penn World Table data constructed by Alan Heston and Robert Summers, online at http://www.nber.org.

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Box 4.7-- An Increase in Population Growth: An Example

Consider an economy in which the parameter α is 1/2--so that the growth multiplier λ = (α/(1–α)) is one--in which the underlying rate of productivity growth g is 1.5% per year, the depreciation rate δ is 3.5% per year, and the savings rate s is 21%. Suppose that the labor force growth rate suddenly and permanently increases from one to two percent per year.

Then before the increase in population growth the steady-state capital output ratio was:

$$\kappa *_{old} = \frac{s}{n_{old} + g + \delta} = \frac{.21}{.01 + .015 + .035} = \frac{.21}{.06} = 3.5$$

After the increase in population growth, the new steady-state capital-output ratio will be:

$$\kappa *_{new} = \frac{s}{n_{new} + g + \delta} = \frac{.21}{.02 + .015 + .035} = \frac{.21}{.07} = 3$$

Before the increase in population growth, the level of output per worker along the old steady-state growth path was:

$$\left(Y_t / L_t\right)_{ss,old} = (\kappa *)^{\lambda} \times E_t = (3.5)^1 \times E_t$$

After the increase in population growth, the level of output per worker along the new steady-state growth path will be:

$$\left(Y_t / L_t\right)_{ss,new} = (\kappa *)^{\lambda} \times E_t = (3.0)^1 \times E_t$$

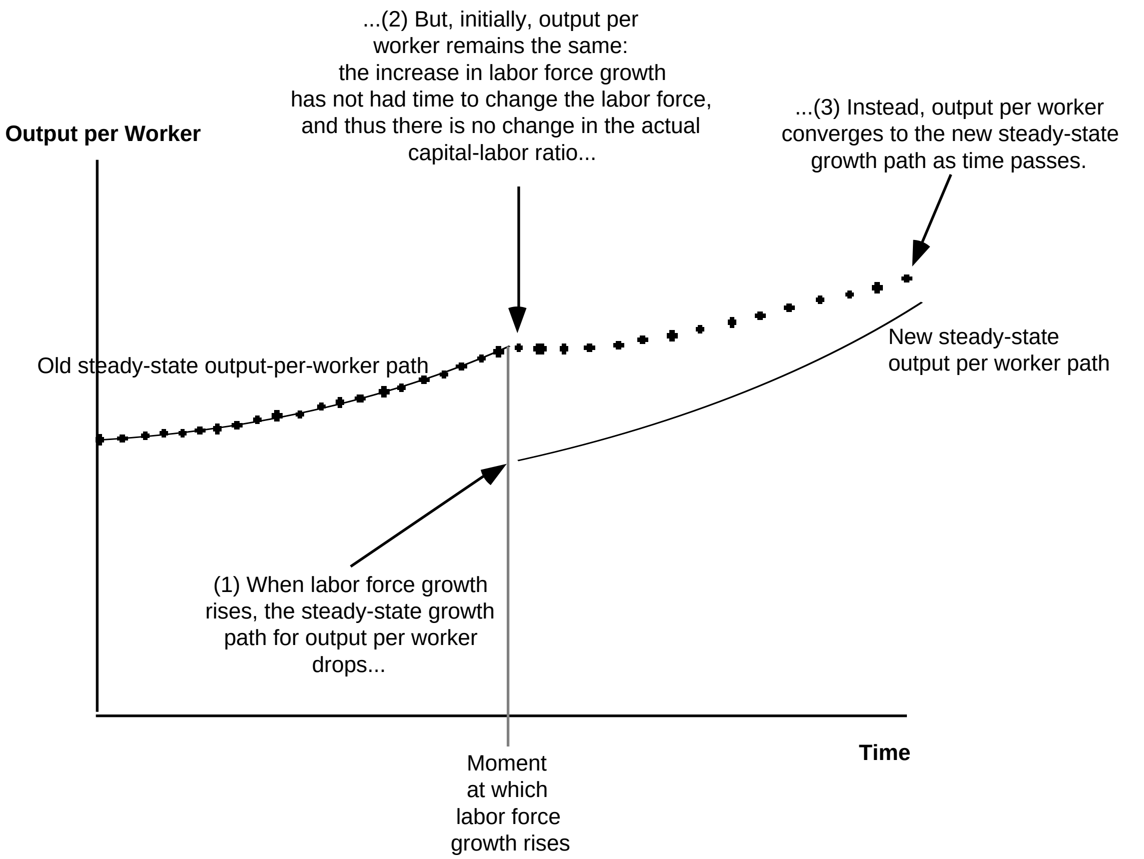
Divide the second of the equations by the first

$$\frac{\left(Y_t / L_t\right)_{ss,new}}{\left(Y_t / L_t\right)_{ss,old}} = \frac{(3.0)^1 \times E_t}{(3.5)^1 \times E_t} = 0.857$$

And discover that output per worker along the new steady-state growth path is only 86% of what it would have been along the old steady-state growth path: faster population growth means that output per worker along the steady-state growth path has fallen by 14 percent.

In the short run this increase in labor force growth will have no effect on output per worker. Just after population growth increases, the increased rate of population growth has had no time to increase the population. It has had no time to affect the actual capital-labor ratio. But over time the economy will converge to the new, lower, steady-state growth path, and output per worker will be reduced by 14% relative to what it would otherwise have been.

Effects of a Rise in Population Growth on the Economy's Growth Path



Legend: Although a sudden change in one of the parameters of the economic growth model causes a sudden change in the location of the economy's steady-state growth path, the economy's level of output per worker does not instantly jump to the new steady-state value. Instead, it converges to the new steady-state value only slowly, over considerable periods of time.

Depreciation and Productivity Growth

Increases or decreases in the depreciation rate will have the same effects on the steady-state capital-output ratio and on output per worker along the steady-state growth path as increases or decreases in the labor force growth rate. The higher the depreciation rate, the lower will be the economy's steady-state capital-output ratio. Why? Because a higher depreciation rate means that the existing capital stock wears out and must be replaced more quickly. The higher the depreciation rate, the larger the share of current investment that must go replace the capital that has become worn-out or obsolete. Thus the lower will be the amount of investment that can be devoted to building up the average ratio of capital to output.

Increases or decreases in the rate of productivity growth will have similar effects as increases or decreases in the labor force growth rate on the steady-state capital-output ratio, but they will have very different effects on the steady-state level of output per worker. The faster the growth rate of productivity, the lower will be the economy's steady-state capital-output ratio. The faster is productivity growth, the higher is output now. But the capital stock depends on what investment was in the past. The faster is productivity growth, the smaller is past investment relative to current production, and the lower is the average ratio of capital to output. So a change in productivity growth will have the same effects on the steady-state capital-output ratio as an equal change in labor force growth.

But a change in productivity growth will have very different effects on output per worker along the steady-state growth path. Output per worker along the steady-state growth path is:

$$\left(Y_t / L_t\right)_{ss} = (\kappa\ *)^{\lambda} \times E_t$$

While an increase in the productivity growth rate g lowers κ*, it increases the rate of growth of the efficiency of labor E, and so in the long run it does not lower but raises output per worker along the steady-state growth path.

The Savings Rate

The higher the share of national product devoted to savings and gross investment, the higher will be the economy's steady-state capital-output ratio. Why? Because more investment increases the amount of new capital that can be devoted to building up the average ratio of capital to output. Double the share of national product spent on gross investment, and you will find that you have doubled the economy's capital intensity--doubled its average ratio of capital to output.

One good way to think about it is that the steady-state capital-output ratio is that at which the economy's investment effort and its investment requirements are in balance. Investment effort is simply s, the share of total output devoted to savings and investment. Investment requirements are the amount of new capital needed to replace depreciated and worn out machines and buildings (a share of total output equal to δ x κ^*), plus the needed to equip new workers who increase the labor force (a share of total output equal to n x κ^*), plus the amount needed to keep the stock of tools and machines at the disposable of workers increasing at the same rate as the efficiency of their labor (a share of total output equal to g x κ^*). So double the savings rate and you double the steady-state capital-output ratio.

Box 4.8--An Increase in the Savings Rate: An Example

For an example of how an increase in savings changes output per worker along the steady-state growth path, consider an economy in which the parameter α is 1/2--so that $\lambda = (\alpha/(1-\alpha))$ is one--in which the underlying rate of labor force growth is 1% per year, the rate of productivity growth g is 1.5% per year, the depreciation rate δ is 3.5% per year. Suppose that the savings rate s was 18%, and suddenly and permanently rises to 24%.

Then before the increase in savings, the steady-state capital output ratio was:

$$\kappa *_{old} = \frac{s_{old}}{n+g+\delta} = \frac{.18}{.01+.015+.035} = \frac{.18}{.06} = 3.0$$

After the increase in savings, the new steady-state capital-output ratio will be:

$$\kappa *_{new} = \frac{s_{new}}{n+g+\delta} = \frac{.24}{.01+.015+.035} = \frac{.24}{.06} = 4$$

Before the increase in savings, the level of output per worker along the old steady-state growth path was:

$$\left(Y_t / L_t\right)_{ss,old} = (\kappa *)^{\lambda} \times E_t = (3.0)^1 \times E_t$$

After the increase in savings, the level of output per worker along the new steady-state growth path will be:

$$\left(Y_t / L_t\right)_{ss,new} = (\kappa *)^{\lambda} \times E_t = (4.0)^1 \times E_t$$

Divide the second of the equations by the first

$$\frac{\left(Y_t / L_t\right)_{ss,new}}{\left(Y_t / L_t\right)_{ss,old}} = \frac{(4.0)^1 \times E_t}{(3.0)^1 \times E_t} = 1.333$$

And discover that output per worker along the new steady-state growth path is 133% of what it would have been along the old steady-state growth path: higher savings means that output per worker along the steady-state growth path has risen by 33 percent.

The increase in savings has no effect on output per worker immediately. Just after the increase in savings has taken place the economy is still on its old, lower steady-state growth path. But as time passes it converges to the new steady-state growth path corresponding to the higher level of savings, and in the end output per worker is 33 percent higher than it would otherwise have been.

How important is all this in the real world? Does a high rate of savings and investment play a role in making countries relatively rich not just in economists' models but in reality? It turns out that it is important indeed, as Figure 4.16 shows. Of the twenty-two countries in the world with GDP per worker levels at least half that of the U.S. level, nineteen have investment shares of more than 20% of output. The high capital-output ratios generated by high investment efforts are a very powerful source of relative prosperity in the world today.

Figure 4.16: National Investment Shares and GDP per Worker Levels

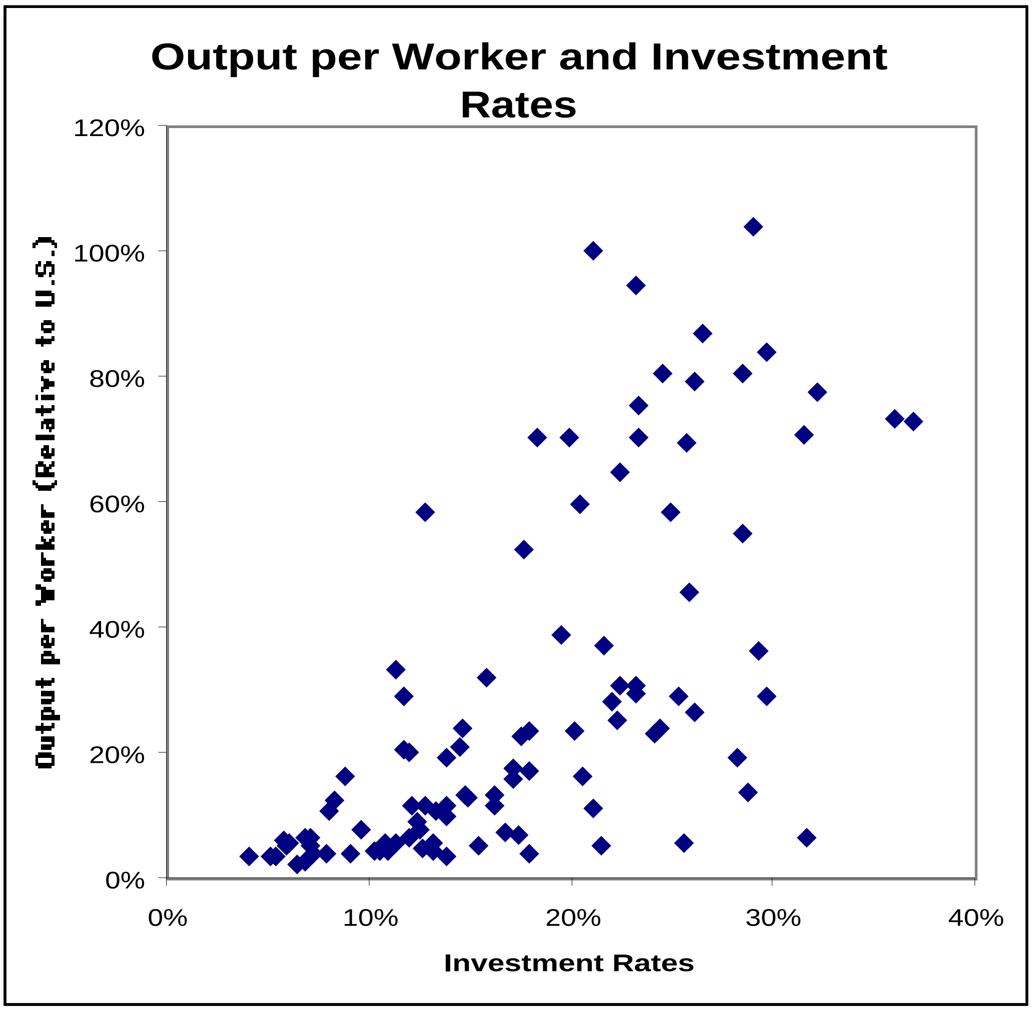


Figure Legend: The average country with an investment share of output of more than twenty-five percent has an output per worker level more than 70% of the U.S. level. The average country with an investment share of output of less than fifteen percent has an output per worker level less than 15% of the U.S. level. Not all of this is due to a one way relationship from a high investment effort to a high steady-state capital-output ratio: countries are not just poor because they invest little, to some degree they invest little because they are poor. But much of it is. High savings and investment rates are a very powerful cause of relative wealth in the world today.

Source: Author's calculations from the Penn World Table data constructed by Alan Heston and Robert Summers, online at http://www.nber.org.

4.4 Chapter Summary

Main Points

1. One principal force driving long-run growth in output per worker is the set of improvements in the efficiency of labor springing from technological progress

2. A second principal force driving long-run growth in output per worker are the increases in the capital stock which the average worker has at his or her disposal and which further multiplies productivity.

3. An economy undergoing long-run growth converges toward and settles onto an equilibrium steady-state growth path, in which the economy's capital-output ratio is constant.

4. The steady -state level of the capital-output ratio is equal to the economy's savings rate, divided by the sum of its labor force growth rate, labor efficiency growth rate, and depreciation rate.

Important Concepts

Production Function

Capital

Output per Worker

Efficiency of Labor

Capital-Output Ratio

Steady-State Growth Path

Savings Rate

Depreciation Rate

Labor Force

Convergence

Consumption per Worker

"Golden Rule" Savings Rate

Analytical Exercises

1. Consider an economy in which the depreciation rate is 3% per year, the rate of population increase is 1% per year, the rate of technological progress is 1% per year, and the private savings rate is 16% of GDP. Suppose that the government increases its budget deficit--which had been at 1% of GDP for a long time--to 3.5% of GDP and keeps it there indefinitely.

What will be the effect of this shift in policy on the economy's steady-state capital-output ratio?

What will be the effect of this shift in policy on the economy's steady state growth path for output per worker? How does your answer depend on the value of the diminishing-returns-to-capital parameter α?

Suppose that your forecast of output per worker 20 years in the future had been $100,000. What is your new forecast of output per worker twenty years hence?

2. Suppose that a country has the production function:

$$Y_t = \left(K_t\right)^{0.5} \times \left(E_t \times L_t\right)^{0.5}$$

What is output Y considered as a function of the level of the efficiency of labor E, the size of the labor force L, and the capital-output ratio (K/Y)?

What is output per worker Y/L?

3. Suppose that with the production function:

$$Y_t = \left(K_t\right)^{0.5} \times \left(E_t \times L_t\right)^{0.5}$$

the depreciation rate on capital is three percent per year, the rate of population growth is one percent per year, and the rate of growth of the efficiency of labor is one percent per year.

Suppose that the savings rate is ten percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path written as a function of the level of the efficiency of labor?

Suppose that the savings rate is fifteen percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path?

Suppose that the savings rate is twenty percent of GDP. What is the steady-state capital-output ratio? What is the value of output per worker on the steady-state growth path?

4. What happens to the steady-state capital-output ratio if the rate of technological progress increases? Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain in the same position?

5. Discuss the following proposition: "An increase in the savings rate will increase the steady-state capital output ratio, and so increase both output per worker and the rate of economic growth in both the short run and the long run."

6. Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain the same if capital were to become more durable--if the rate of depreciation on capital were to fall?

7. Suppose that a sudden disaster--an epidemic, say--reduces a country's population and labor force, but does not affect its capital stock. Suppose further that the economy was on its steady-state growth path before the epidemic.

What is the immediate effect of the epidemic on output per worker?

On the total economy-wide level of output?

What happens subsequently?

8. According to the marginal productivity theory of distribution, in a competitive economy the rate of return on a dollar's worth of capital--its profits or interest--is equal to capital's marginal productivity. With the production function:

$$\left(\frac{Y_t}{L_t}\right) = \left(\frac{K_t}{L_t}\right)^{\alpha} (E_t)^{1-\alpha}$$

what is the marginal product of capital? How much is total output (Y, not Y/L) boosted by the addition of an extra unit to the capital stock?

9. According to the marginal productivity theory of distribution, in a competitive economy the rate of return on a dollar's worth of capital--its profits or interest--is equal to capital's marginal productivity. If this theory holds and the marginal productivity of capital is indeed:

$$dY/dK = \alpha \text{ x } (Y/K)$$

How large are the total earnings received by capital? What share of total output will be received by the owners of capital as their income?

10. Suppose that environmental regulations lead to a slowdown in the rate of growth of the efficiency of labor in the production function, but also lead to better environmental quality. Should we think of this as a “slowdown” in economic growth or not?

Policy-Relevant Exercises

1. In the mid-1990s during the Clinton Presidency the U.S. eliminated its federal budget deficit. The national savings rate was thus boosted by 4% of GDP, from 16% to 20% of real GDP. In the U.S. in the mid-1990s, the rate of labor force growth was 1% per year, the depreciation rate was 3% per year, the rate of increase of the efficiency of labor was 1% per year, and that the diminishing-returns-to-capital parameter α is 1/3. Suppose that these rates continue into the indefinite future.

   Suppose that the federal budget deficit had remained at 4% indefinitely. What then would have been the U.S. economy's steady-state capital-output ratio? If the efficiency of labor in 2000 were $30,000 per year, what would have been your forecast of output per worker in the U.S. in 2040?

   After the elimination of the federal budget deficit, what would be your calculation of been the U.S. economy's steady-state capital-output ratio? If the efficiency of labor in 2000 were $30,000 per year, what would have been your forecast of output per worker in the U.S. in 2040?

2. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 1/2, and if your estimate of the efficiency of labor in 2000 were not $30,000 but $15,000 a year?

3. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 2/3?

4. What are the long-run costs as far as economic growth is concerned of a policy of taking money that would reduce the national debt—and thus add to national savings—and distributing it as tax cuts instead? What would be the long-run benefits of such a policy? How could we decide whether such a policy was a good thing or not?

5. At the end of the 1990s it appeared that because of the computer revolution the rate of growth of the efficiency of labor in the United States had doubled, from 1 percent per year to 2 percent per year. Suppose this increase were to be permanent. And suppose the rate of labor force growth were to remain constant at 1 percent per year, the depreciation rate were to remain constant at 3 percent per year, and the American savings rate (plus foreign capital invested in America) were to remain constant at 20 percent per year. Assume that the efficiency of labor in the U.S. in 2000 is $15,000 per year, and that the diminishing-returns-to-capital parameter α is 1/3.

What is the change in the steady-state capital-output ratio? What is the new capital-output ratio?

What would such a permanent acceleration in the rate of growth of the efficiency of labor change your forecast of the level of output per worker in 2040?

6. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 1/2, and if your estimate of the efficiency of labor in 2000 were not $30,000 but $15,000 a year?

7. How would your answers to the above question change if your estimate of the diminishing-returns-to-capital parameter α were not 1/3 but 2/3?

8. Output per worker in Mexico in the year 2000 is about $10,000 per year. Labor force growth is 2.5% per year. The depreciation rate is 3% per year. The rate of growth of the efficiency of labor is 2.5% per year. The savings rate is 16% of GDP. And the diminishing-returns-to-capital parameter α is 0.5.

What is Mexico's steady-state capital-output ratio?

Suppose that Mexico today is on its steady-state growth path. What is the current level of the efficiency of labor E?

What is your forecast of output per worker in Mexico in 2040?

9. In the framework of the question above…

…how much does your forecast of output per worker in Mexico in 2040 increase if Mexico's domestic savings rate remains unchanged but it is able to finance extra investment equal to 4% of GDP every year by borrowing from abroad?

…how much does your forecast of output per worker in Mexico in 2040 increase if the labor force growth rate immediately falls to 1% per year?

…how much does your forecast of output per worker in Mexico in 2040 increase if both happen?

10. Consider an economy with a labor force growth rate of 2% per year, a depreciation rate of 4% per year, a rate of growth of the efficiency of labor of 2% per year, and a savings rate of 16% of GDP.

Suppose that the diminishing-returns-to-capital parameter α is 1/3. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?

Suppose that the diminishing-returns-to-capital parameter α is 1/2. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?

Suppose that the diminishing-returns-to-capital parameter α is 2/3. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?

Suppose that the diminishing-returns-to-investment capital α is 3/4. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?