The "Golden Rule"

Choosing a National Savings Rate

What can we say about economic policy and long-run growth? To keep matters simple, let us assume that the government can—by proper fiscal and monetary policies—set and keep the economy's savings-investment rate s at whatever level it wishes. What level should the government choose for the economy's savings rate?

It seems reasonable to assume that the government's objective is to maximize the well-being of the individuals who make up the society by maximizing the amount of goods and services that they can consume. Let us, for the moment, simplify things further and say that consumption C is equal to total production Y minus investment I:

$$C_t = Y_t - I_t$$

Where investment I is equal to the savings rate s times total production Y:

$$I_t = s \times Y_t$$

So consumption per worker C/L is equal to:

$$\frac{C_t}{L_t} = (1-s)Y_t$$

If we focus our attention on steady-states only, steady-state consumption per worker on the long-run growth path is equal to:

$$\frac{C_t}{L_t} = (1-s)\left(\frac{s}{n+g+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} \times E_t$$

Maximizing Steady-State Consumption per Worker

What level of the savings rate s should the government choose if it wishes the economy to be on the long-run growth path that has the highest level of consumption per worker? If we look at the rate of change—the derivative—of consumption per worker as a function of the savings rate:

$$\frac{d}{ds}\left(\frac{C_t}{L_t}\right) = \left(-(s)^{\left(\frac{\alpha}{1-\alpha}\right)} + \left(\frac{\alpha}{1-\alpha}\right)(1-s)(s)^{\left(\frac{2\alpha-1}{1-\alpha}\right)}\right) \times \left(\frac{E_t}{(n+g+\delta)^{\left(\frac{\alpha}{1-\alpha}\right)}}\right)$$

$$\frac{d}{ds}\left(\frac{C_t}{L_t}\right) = \left(-1 + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-s}{s}\right)\right) \times (s)^{\left(\frac{\alpha}{1-\alpha}\right)}\left(\frac{E_t}{(n+g+\delta)^{\left(\frac{\alpha}{1-\alpha}\right)}}\right)$$

The rate of change of consumption per worker is zero—and the level of consumption per worker is at its highest—when:

$$s = \alpha$$

This savings rate is called the “golden rule” savings rate. The associated steady-state growth path is called the “golden rule” steady-state growth path.

The Golden Rule and the Marginal Product of Capital

Another way to look at the golden rule steady-state is to look at the marginal product of capital—the amount by which an additional unit of capital boosts output. The marginal product of capital is:

$$\frac{dY}{dK} = \alpha \frac{Y}{K}$$

Which is, at the steady-state growth path with K/Y = s/(n+g+δ) = α/(n+g+δ), equal to:

$$\frac{dY}{dK} = \alpha \frac{(n+g+\delta)}{\alpha} = n + g + \delta$$

At the golden rule steady-state growth path, the marginal product of capital is equal to the sum of the population growth rate, the efficiency of labor growth rate, and the depreciation rate.

To make the point another way, suppose that the economy starts with some steady-state capital-output ratio κ' and the government considers taking steps to increase the savings rate to boost the capital stock by one unit. The amount by which the change increases production is the marginal product of capital—n+g+δ. But this increase in the capital stock increases the amount of savings needed to maintain the new, higher capital-output ratio: n is needed to keep up with population growth, g to maintain pace with the increased efficiency of labor, and δ to offset depreciation on the higher capital stock.

Thus when the marginal product of capital is equal to n+g+δ—and the savings-investment rate s is equal to α—then the extra output produced by an increase in the capital-output ratio is just equal to the increase in savings and investment required to maintain that extra increase in the capital-output ratio. When the capital-output ratio is less than the golden rule steady-state, an increase in the capital-output ratio raises output by more than the required increase in savings and investment: thus consumption per worker can increase. When the capital-output ratio is more than the golden rule steady-state, an increase in the capital-output ratio does not raise output by enough to offset the required increase in savings and investment: thus consumption per worker must fall.

Implications for Economic Policy

If an economy begins at a steady state with a higher capital-output ratio than the golden rule steady state, then consumption per worker can be increased by reducing the savings rate. A decline in the savings rate will boost the steady-state level of consumption per worker, and thus boost consumption per worker in the long run. Moreover, by cutting back on savings and increasing the funds available for consumption, consumption per worker can be increased in the short run as well.

If the economy begins at a steady state with a lower capital-output ratio than in the golden rule, then the government must take steps to raise the savings rate in order to reach the golden rule steady state. In the long run, this increase in the savings rate will boost the steady-state level of consumption per worker, and thus boost consumption per worker in the long run. However, the increase in the savings rate reduces the funds available for consumption in the short run. When the economy begins above the golden rule, reaching the golden rule produces higher consumption at all moments in time. But when the economy begins below the golden rule, reaching the golden rule requires reducing the level of consumption now and in the near future in order to boost consumption in the long run.

A government trying to consider whether to try to move the economy toward the golden rule steady state has to consider whether the long run boost to consumption outweighs the short run cut in consumption. The government must decide whether this tradeoff between the near future and the distant future is worthwhile.