A Model of Human Capital and Growth:

Assumptions:

$$Y_t = K_t^{\alpha} H_t^{\beta} \left[A_t L_t\right]^{1-\alpha-\beta}$$

$$\frac{dK_t}{dt} = s_K Y_t$$

$$\frac{dH_t}{dt} = s_H Y_t$$

$$\frac{dA_t}{dt} = gA_t$$

$$\frac{dL_t}{dt} = nL_t$$

Intensive form:

$$y_t = k_t^{\alpha} h_t^{\beta}$$

$$\frac{dk_t}{dt} = s_K y_t - (n+g)k_t$$

$$\frac{dh_t}{dt} = s_H y_t - (n+g)h_t$$

Finding k-constant and h-constant locuses:

$$\frac{dk_t}{dt} = s_K y_t - (n+g)k_t = s_K k_t^{\alpha} h_t^{\beta} - (n+g)k_t$$

$$0 = s_K k_t^{\alpha} h_t^{\beta} - (n+g)k_t$$

$$(n+g)k_t = s_K k_t^{\alpha} h_t^{\beta}$$

$$k_t^{1-\alpha} = \frac{s_K h_t^{\beta}}{n+g}$$

$$k_t = \left[\frac{s_K}{n+g}\right]^{\frac{1}{1-\alpha}} h_t^{\frac{\beta}{1-\alpha}}$$

$$h_t = \left[\frac{s_H}{n+g}\right]^{\frac{1}{1-\beta}} k_t^{\frac{\alpha}{1-\beta}}$$

And the point at which the curves given by these last two equations intersect is the economy's attractor:

$$k^* = \frac{s_K^{\frac{1-\beta}{1-\alpha-\beta}} s_H^{\frac{\beta}{1-\alpha-\beta}}}{(n+g)^{\frac{1}{1-\alpha-\beta}}}$$

$$h^* = \frac{s_H^{\frac{1-\alpha}{1-\alpha-\beta}} s_K^{\frac{\alpha}{1-\alpha-\beta}}}{(n+g)^{\frac{1}{1-\alpha-\beta}}}$$

$$y^* = \frac{s_K^{\frac{\alpha}{1-\alpha-\beta}} s_H^{\frac{\beta}{1-\alpha-\beta}}}{(n+g)^{\frac{\alpha+\beta}{1-\alpha-\beta}}}$$

$$z_K{}^* = \frac{s_K}{n+g}$$

$$z_H{}^* = \frac{s_H}{n+g}$$

$$y^* = k^{*\alpha}\, h^{*\beta} = \left[z^*_K\; y^*\right]^{\alpha} \left[z_H{}^* y^*\right]^{\beta}$$

$$y^* = \left[z^*_K\right]^{\frac{\alpha}{1-\alpha-\beta}} \left[z^*_H\right]^{\frac{\beta}{1-\alpha-\beta}}$$

$$r^*_K = \alpha \frac{n+g}{s_K}$$

$$r^*_H = \beta \frac{n+g}{s_H}$$

$$\frac{dz_{Kt}}{dt} = (1-\alpha)s_K - \beta s_H \frac{z_{Kt}}{z_{Ht}} - (1-\alpha-\beta)(n+g)z_{Kt}$$

$$\frac{dz_{Ht}}{dt} = (1-\beta)s_H - \alpha s_K \frac{z_{Ht}}{z_{Kt}} - (1-\alpha-\beta)(n+g)z_{Ht}$$

If we have been on a path such that the ratio of z(K)/z(H) is at its steady-state value: z(K)/z(H) = s(K)/s(H), then:

$$\frac{dz_{Kt}}{dt} = (1-\alpha-\beta)s_K - (1-\alpha-\beta)(n+g)z_{Kt}$$

$$\frac{dz_{Ht}}{dt} = (1-\alpha-\beta)s_H - (1-\alpha-\beta)(n+g)z_{Ht}$$

z converges z* at dying exponential rate:

$$\lambda = (1-\alpha-\beta)(n+g)$$