Economics 202a; Problem Set 4

Due Thursday, April 3

(From Romer's *Advanced Macroeconomics*, problems 4.4, 4.8, and 4.15)

1. Suppose that a representative individual's period-t utility function is:

$$u_t = \ln(c_t) + \frac{b(1-\ell_t)^{1-\gamma}}{1-\gamma}$$

a. Suppose that the representative individual "lives" for just one period, and that his/her budget constraint is:

$$c_t = w_t \ell_t$$

Solve for labor supply and consumption as functions of the real wage w_t.

b. Suppose that the representative individual "lives" for just two periods, so that the lifetime budget constraint is:

$$c_{1,t} + \frac{c_{2,t+1}}{1+r_{t+1}} = w_t \ell_{1,t} + \frac{w_{t+1}\ell_{2,t+1}}{1+r_{t+1}}$$

and the lifetime utility function is:

$$U = u_{1,t} + e^{-\rho} u_{2,t+1}$$

Solve for labor supply and consumption in both periods as functions of the real wages in periods t and t+1. How does the relative demand for leisure in the two periods depend on the relative wage? How does it depend on the interest rate? Can you explain why changing γ changes the responsiveness of labor supply to wages and the interest rate?

2. Consider an economy with a constant population of infinitely-lived individuals. The representative individual maximizes the expected value of:

$$U = \sum_{t=0}^{\infty} \frac{C_t - \theta C_t^2}{(1+\rho)^t}$$

with θ greater than zero. Assume that consumption always remains within the range in which the marginal utility of consumption is positive. Assume further that output is linear in capital plus a disturbance:

$$Y_t = AK_t + e_t$$

$$e_t = \phi e_{t-1} + \varepsilon_t$$

where the ε's are mean-zero, independent and identically distributed shocks. Assume, last, that the capital stock evolves according to:

$$K_{t+1} = K_t + Y_t - C_t$$

a. What is the interest rate in this model?

b. Assume that the rate of time discount ρ=A. Find the first-order condition [Euler equation] relating consumption now and expectations of consumption next period.

c. Still assuming that the rate of time discount ρ=A, suppose that a small bird tells you that consumption takes the form:

$$C_t = \alpha + \beta K_t + \gamma e_t$$

Find the values of the parameters α, β, and γ that satisfy the consumption Euler equation for all possible values of K and e.

d. Find the impulse response functions giving the effects over time of a one-time shock ε_t on the time paths of Y, K, and C.

3. Consider the production function and the equation of motion for the capital stock in our canonical real business cycle model:

$$K_{t+1} = (1-\delta)K_t + (Y_t - C_t - G_t)$$

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

a. Substitute the production function into the equation of motion for the capital stock, and find:

$$\frac{\partial \ln(K_{t+1})}{\partial \ln(K_t)}$$

as a function of today's and next period's capital stock, and of the--net of depreciation--real interest rate:

$$r_{t+1} = \frac{\partial Y_{t+1}}{\partial K_{t+1}} - \delta$$

b. Evaluate

$$\frac{\partial \ln(K_{t+1})}{\partial \ln(K_t)}$$

at the economy's balanced growth path, and show that it is equal to:

$$\left.\frac{\partial \ln(K_{t+1})}{\partial \ln(K_t)}\right|_{K_t = K_t^*, K_{t+1} = K_{t+1}^*} = \frac{1+r^*}{e^{n+g}}$$

c. Recognize that along the balanced growth path:

$$Y_t^* = \frac{(r^* + \delta)K_t^*}{\alpha}$$

$$C_t^* = Y_t^* - G_t^* + (1-\delta)K_t^* - K_{t+1}^* = Y_t^* - G_t^* + (1 - e^{n+g} - \delta)K_t^*$$

Use part (b) to derive a first order Taylor-series approximation (linear in the logs of the variables) to the capital stock equation of motion.

d. Rewrite your answer in part (c) to express the log-deviation of next period's capital stock from the balanced growth path as a function of this period's log-deviations of capital, technology, labor supply, government spending, and consumption from their balanced growth paths.

e. If your log-linearization turned out to imply that:

$$\tilde{K}_{t+1} = \frac{1+r^*}{e^{n+g}}(\tilde{K}_t - \tilde{C}_t) + \frac{(1-\alpha)(r^*+\delta))}{\alpha e^{n+g}}(\tilde{A}_t + \tilde{L}_t - \tilde{C}_t) - \frac{(r^*+\delta)(G_t^*/Y_t^*)}{\alpha e^{n+g}}(\tilde{G}_t - \tilde{C}_t) + \tilde{C}_t$$

congratulate yourself. If not, look back and see if you can guess where your approximations went wrong.