

Problem Set 3, Economics 202, Fall 1998

New Growth Models

1. Suppose we have a model economy:

$$Y_t = A_t(1 - a_L)L_t$$

$$\frac{dA_t}{dt} = B[a_L L_t]^\gamma A_t^\theta$$

$$\frac{dL_t}{dt} = nL_t$$

and that $\theta < 1$. Derive an expression for A_t on the economy's balanced growth path as a function of B , a_L , γ , θ , and L_t . Can you determine the value of a_L that maximizes output on the balanced growth path?

2. Suppose we have the model economy:

$$Y_t = [(1 - a_K)K_t]^\alpha [A_t(1 - a_L)L_t]^{1-\alpha}$$

$$\frac{dA_t}{dt} = B[a_K K_t]^\beta [a_L L_t]^\gamma [A_t]^\theta$$

$$\frac{dK_t}{dt} = sY_t$$

$$\frac{dL_t}{dt} = nL_t$$

with positive population growth, and with $\alpha + \beta < 1$ so that a balanced growth path exists.

Use the technology-growth capital-accumulation phase diagram to *qualitatively* analyze the effects of:

- (a) an increase in n
- (b) an increase in a_K
- (c) an increase in θ

3. Suppose we have the model economy:

$$Y_t = [(1 - a_K)K_t]^\alpha [A_t(1 - a_L)L_t]^{1-\alpha}$$

$$\frac{dA_t}{dt} = B[a_K K_t]^\beta [a_L L_t]^\gamma [A_t]^\theta$$

$$\frac{dK_t}{dt} = s Y_t$$

$$\frac{dL_t}{dt} = n L_t$$

with positive population growth, and with $\theta < 1$ so that a balanced growth path exists. Suppose that the economy is initially on its balanced growth path, and that there is then a permanent increase in s :

(a) How does this permanent increase in s affect the location of the steady-state on the technology-growth capital-accumulation phase diagram? How does it affect the location of the economy on the phase diagram at the moment of change?

(b) What are the dynamics of the growth rates of technology and capital after the increase in s ?

(c) Sketch the path of output per worker after the change, compared to what would have occurred had the change not taken place.

4. Suppose that we have the model economy:

$$Y_t = K_t^\alpha H_t^\beta [A_t L_t]^{1-\alpha-\beta}$$

$$\frac{dK_t}{dt} = s_K Y_t$$

$$\frac{dH_t}{dt} = s_H Y_t$$

$$\frac{dA_t}{dt} = g A_t$$

$$\frac{dL_t}{dt} = n L_t$$

(a) What is consumption per unit of effective labor on the balanced growth path?

(b) What values of the physical capital savings rate and the human capital savings rate maximize consumption per unit of effective labor?