How Fast Should an Economy Head for the "Golden Rule"? Some Simple Math

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Econ 101b

Begin with an objective function—social-welfare or representative-agent utility, as a function of per-capita consumption over time:

$$\max \int_{t=0}^{\infty} e^{-\rho t}\left(\frac{c_t^{1-\theta}-1}{1-\theta}\right)$$

Two parameters: θ and ρ.

Consider increasing savings and then decreasing it in order to postpone a small amount of consumption for a small period of time, and then returning to the previously-scheduled path for consumption. Then we have, at the optimal plan:

$$-(\delta c)\left(c_t^{-\theta}\right)+(\delta c)\left(1+r(\delta t)\right)\left(e^{-\rho(\delta t)}\right)\left(c_{t+\delta t}^{-\theta}\right)=0$$

Solve the algebra:

$$\left(1+r(\delta t)\right)\left(e^{-\rho(\delta t)}\right)\left(\frac{c_{t+\delta t}^{-\theta}}{c_t^{-\theta}}\right)=1$$

$$\left(1+r(\delta t)\right)\left(e^{-\rho(\delta t)}\right)\left(\frac{c_t+(\delta t)\frac{dc_t}{dt}}{c_t}\right)^{-\theta}=1$$

$$\left(1+r(\delta t)\right)\left(1-\rho(\delta t)\right)\left(1+\frac{1}{c_t}\frac{dc_t}{dt}(\delta t)\right)^{-\theta}=1$$

$$\left(1+r(\delta t)\right)\left(1-\rho(\delta t)\right)\left(1-\theta\frac{1}{c_t}\frac{dc_t}{dt}(\delta t)\right)=1$$

And arrive at:

$$r - \rho = \frac{\theta}{c_t}\frac{dc_t}{dt}$$

This tells us that the government's fiscal policy should be to adjust the savings rate so that per-capita consumption grows according to:

$$\frac{dc_t}{dt} = \frac{(r - \rho)c_t}{\theta}$$

What is r? r is the **social** marginal product of capital:

$$r_t = \frac{\partial Y_t}{\partial K_t} = \frac{\alpha Y_t}{K_t}$$

So the right thing for the government to do is to change spending and taxes until consumption is growing at a rate $(r-\rho)/\theta$.

- What is r? 6% + 3% taxes + 3% labor rents + 3% externalities = 15%…
- What is ρ? Impatience? Or should ρ be $-n$? And what if we recognize that n is endogenous? There are unsolved problems in the theory of applied utilitarianism…
- What is θ? θ is somewhere between one and 3…